# 97. On the Šilov Boundaries of Function Algebras

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Let $C(X)$ be the algebra of all complex-valued continuous functions on a compact Hausdorff space $X$. By a *function algebra* we mean a closed (by supremum norm) subalgebra in $C(X)$ containing constants and separating points of $X$. For $F\subset X$, let $f|F$ be the restriction of the function $f$ to $F$ and $A|F=\{f|F; f\in A\}$. We easily see that for any closed subset $F$ containing the Šilov boundary $\partial A$ of $A$ (cf. [1], [6]), $A|F$ is closed in $C(F)$. Conversely, it is natural to raise the following question: Let $F_0$ be a closed subset in $X$ and let $A|F$ be closed in $C(F)$ for any closed subset $F$ containing $F_0$. Then, does $F_0$ contain $\partial A$? The main purpose of this note is to answer the question under certain conditions (Theorems 1 and 2). The proof of Theorem 1 is a modification of that of Glicksberg's theorem (cf. [3]) and we obtain the Glicksberg's theorem as a corollary.

Let $A$ be a function algebra on $X$. Then there is a unique minimal closed subset $E$ of $X$ such that any continuous function zero on $E$ is in $A$. This closed subset $E$ is called the *essential set* of $A$. $A$ is an *essential algebra* if the essential set of $A$ is $X$ (cf. [2]). A function algebra $A$ is said to be an *antisymmetric algebra* (or an *analytic algebra*) if any real-valued function in $A$ is always constant (or any function in $A$ vanishing on a non-empty open set in $X$ is always identically zero) (cf. [4]). An analytic algebra is antisymmetric and an antisymmetric algebra is an essential algebra (cf. [4]).

Our main theorem is the following

**Theorem 1.** *Let $A$ be an essential algebra and let $F_0$ be a closed subset in $X$. If $A|F$ is closed in $C(F)$ for any closed subset $F$ containing $F_0$, then $F_0$ contains the Šilov boundary $\partial A$ of $A$.**)

*Proof.* We set first $F_1=\{y|y\in X, |f(y)|\leqq \sup_{x\in F_0}|f(x)| \text{ for any } f\in A\}$. Then we see that $F_1$ is a closed set in $X$ containing $F_0$. If $F_1=X$, then $\sup_{x\in F_0}|f(x)|=\sup_{x\in F_1}|f(x)|=\sup_{x\in X}|f(x)|$ for any $f\in A$, so $F_0\supset\partial A$. Therefore, in order to prove the theorem we need only to show that $F_1=X$. Suppose the contrary: $X\neq F_1$. We can show first that there is a function $f\in A$ such that $f(x)=1$ on $P$ and $f(x)=0$ on $Q$ for any closed set $P$ and for any closed set $Q$ with $Q\supset F_1$, $P\frown Q=O$. For, let $p, q$

*) After this paper had been accepted for publication, Prof. I. Glicksberg informed me that this theorem can be also proved by direct use of his theorem [3].