92. A Property of Certain Differentiable Manifolds

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Let M be a compact oriented differentiable manifold without boundary which satisfies the following conditions:

(1) M is $(n-1)$-connected,

(2) $\dim M=2n+1$, $n\equiv 0 \pmod 2$.

Then the oriented cobordism class of M is determined by a Stiefel-Whitney number $W_n\cdot W_{n+1}[M]$, for other Stiefel-Whitney numbers and all Pontryagin numbers vanish. In this paper we shall show that the property of M to be cobordant to zero can be represented by a property of $H_n(M, Z)$. In case $n=2$ this was done by Wall [1].

We shall prove the following

Theorem. $W_n\cdot W_{n+1}[M]=0 \Longleftrightarrow H_n(M, Z)\approx F\oplus T\oplus T$

$W_n\cdot W_{n+1}[M]\neq 0 \Longleftrightarrow H_n(M, Z)\approx F\oplus T\oplus T\oplus Z_2$

where F and T denote a free abelian group and a torsion group respectively and Z_2 is the group of order 2, $\oplus$ denotes the direct sum.

The proof will be given in several steps.

$H_n(M, Z)$ can be decomposed as follows:

$$H_n(M, Z)=\sum_{i=1}^{a_0} Z[\bar{u}_i]+\sum_p\sum_i\sum_{j=1}^{a_i(p)} Z_{p^i}[\bar{u}_p^{i,j}],$$

where p runs over all prime numbers, $\bar{u}_i$, $\bar{u}_p^{i,j}$ denote generators. Since we are interested in $a_i(p)$, it is sufficient for us to consider $H^n(M, Z_p)$ and $H^{n+1}(M, Z_p)$, i.e.

$$H^n(M, Z_p)=\sum_{i=1}^{a_0} Z_p[u_i]+\sum_{i=1}\sum_{j=1}^{a_i(p)} Z_{p^i}[u_p^{i,j}]$$

$$H^{n+1}(M, Z_p)=\sum_{i=0}^{a_0} Z_p[v_i]+\sum_{i=1}\sum_{j=1}^{a_i(p)} Z_{p^i}[v_p^{i,j}].$$

Now we consider a matrix $A=(a_{s,t})$ over Z_p defined by $a_{s,t}=u_p^{j,l}\cdot v_p^{i,k}[M]$ for $a_0+\sum_{m=1}^{j-1}a_m(p)<s\leqq a_0+\sum_{m=1}^{i}a_m(p)$, $a_0+\sum_{m=1}^{i-1}a_m(p)<t\leqq a_0+\sum_{m=1}^{i}a_m(p)$ where j, l, i, k are given by $s=a_0+\sum_{m=1}^{j-1}a_m(p)+l$, $t=a_0+\sum_{m=1}^{i-1}a_m(p)+k$, $i, j\geqq 1$, and $a_{s,t}=u^s\cdot v^t[M]$ for $1\leqq s, t\leqq a_0$.

By Poincaré duality we have $\det A\neq 0$. Let Δ_p^i denote the higher Bockstein operator. As we can take $v_p^{i,k}=\Delta_p^i(u_p^{i,k})$ $(i\geqq 1)$, we obtain

Lemma 1. If p is odd, we have

(1) $u^k\cdot v_p^{i,j}=0$

(2) $u_p^{i,j}\cdot v_p^{s,t}=0$ $(s<i)$