127. Some Applications of the Functional-Representations of Normal Operators in Hilbert Spaces. IX

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In this paper we shall discuss, under some conditions, the relation between the distribution of ζ-points of the function $S(\lambda)$ defined in the statement of Theorem 1 [cf. Proc. Japan Acad., Vol. 38, 263–268 (1962)] and that of ζ-points of the ordinary part $R(\lambda)$ of $S(\lambda)$, on the supposition that $R(\lambda)$ is a polynomial in λ of degree less than or equal to d.

Theorem 23. Let $S(\lambda)$ and $\{\lambda_\nu\}$ be the same notations as those defined in the statement of Theorem 1; let the ordinary part $R(\lambda)$ of $S(\lambda)$ be a polynomial in λ of degree less than or equal to d; let α be one of ζ-points of $R(\lambda)$ for an arbitrarily given complex number ζ; let ρ and μ be arbitrarily prescribed positive numbers satisfying the conditions $\sup_\nu|\lambda_\nu|<\rho<\infty$ and $0<\mu<1$ respectively; let r be a positive number such that $\frac{\rho}{\mu}\leqq r<\infty$; let $m_R(r,\alpha)$ denote the minimum of the modulus $|R(\lambda)-\zeta|$ on the circle $|\lambda-\alpha|=r$; and let $K=\frac{M_s(\rho,0)}{(1-\mu)\rho^d}$ where $M_s(\rho,0)$ denotes the maximum of the modulus $|S(\lambda)|$ on the circle $|\lambda|=\rho$. If

$$|\alpha|>\frac{Kr^{d+1}}{m_R(r,\alpha)}+2r,$$

then, in the interior of the circle $|\lambda-\alpha|=r$, $S(\lambda)$ has ζ-points whose number (counted according to multiplicity) is equal to that of ζ-points of $R(\lambda)$ in the interior of the same circle.

Proof. Since, by hypotheses, $R(\lambda)$ is a polynomial in λ of degree less than or equal to d,

$$M_s\left(\frac{\rho}{\kappa},0\right)\leqq\frac{M_s(\rho,0)}{(1-\mu)\kappa^d}\quad(0<\kappa\leqq\mu),$$

as we have already shown in the course of the proof of Theorem 13 stated in Part V [1]. Substituting $\frac{M_s(\rho,0)}{1-\mu}$ in this inequality by $K\rho^d$, we have therefore

$$M_s(r,0)\leqq Kr^d\quad\left(\frac{\rho}{\mu}\leqq r<\infty\right). \tag{23}$$

Moreover, if we denote by $\chi(\lambda)$ the sum of the first and the second principal parts of $S(\lambda)$,