145. A New Algebraical Property of Certain von Neumann Algebras

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1. It has been a subject since F. J. Murray and J. von Neumann [5] that there are two non-hyperfinite, non-isomorphic, continuous finite factors. Recently, J. Schwartz [7] has affirmatively solved the subject by introducing the Property P (Definition 1 below). However, the property P is spatial, and a question still remains to find that a purely algebraical property can serve his need.

In the present note, we shall introduce a purely algebraical property, the property Q (Definition 2), and show that the property Q is sufficient to serve Schwartz' need. Actually, we shall show that the property Q implies the property P in Theorem 1, and that the properties P and Q are equivalent for a group operator algebra in Theorem 2. Besides, we shall show directly that the hyperfinite continuous factor satisfies the property Q in Theorem 5. Furthermore, we shall show that the tensor product of two von Neumann algebras having the property Q satisfies also the property Q in Theorem 6.

2. Let G be a (discrete) group. Let $L^\infty(G)$ be the algebra of all bounded complex-valued functions defined on G. A functional $\int x(g)dg$ on $L^\infty(G)$ will be called a *Banach mean*, cf. [3], when it has the following properties: For $x, y \in L^\infty(G)$ and $g, h \in G$,

1° $\int[\alpha x(g)+\beta y(g)]dg=\alpha\int x(g)dg+\beta\int y(g)dg,$

2° $\int x(g)dg \geqq 0$ if $x(g)\geqq 0$ for all $g \in G$,

3° $\int x(gh)dg=\int x(g)dg,$

4° $\int 1\ dg=1,$

where α and β are complex numbers. According to Day [3], if G has a Banach mean, G will be called an *amenable group*. If $\{T_g \mid g\in G\}$ is a uniformly bounded family of operators on a Hilbert space, then there exists a finite constant K with $|(T_g x \mid y)|\leqq K||x||\cdot||y||$. Hence $[x\mid y]=\int(T_g x\mid y)dg$ is a bounded conjugate bilinear form on the Hilbert space. Consequently, there exists a unique bounded operator T such that $[x\mid y]=(Tx\mid y)$. We shall call T the *operator Banach mean* on G and write it by $T=\int T_g dg$. A similar construction is