138. *The Relativity Theory in the Einstein Space under the Extended Lorentz Transformation Group*

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The general theory of relativity of A. Einstein was based on the non-definite quadratic differential form

(1) $$dS^2 = g_{\mu\nu}(x^\sigma)\, dx^\mu dx^\nu, \quad (\lambda, \mu, \nu, \cdots = 1, 2, 3, 4)$$

and grasped as the *Riemannian geometry of the Einstein space*:

(i) $$R_{\mu\nu} = 0,$$ (ii) $$R_{\mu\nu} = \frac{R}{4} g_{\mu\nu},$$

the path of a free particle being the geodesic curve:

(2) $$\frac{d^2x^\lambda}{dS^2} + \begin{Bmatrix} \lambda \\ \mu\nu \end{Bmatrix} \frac{dx^\mu}{dS}\frac{dx^\nu}{dS} = 0.$$

The fundamental assumption was the so-called *principle of equivalence.* The merit was the *geometrization of physics.* But the demerit was the obscurity of the physical side caused by the laborious calculations in terms of $g_{\mu\nu}$ and $\begin{Bmatrix} \lambda \\ \mu\nu \end{Bmatrix}$ as well as by too much forcing physical interpretations. Thus the Einstein's theory has remained merely as a *conjecture* for the last 47 years without becoming a decisive immortal theory.

With the hope to make it a decisive theory comparable with the Newton's theory, the present author ([1]–[14]) started with the expressibility of (1) in the form

(3) $$dS^2 = g_{\mu\nu} dx^\mu dx^\nu = (-1)^{1+\delta^4_l} \omega^l \omega^l, \quad (\omega^l = \omega^l_\mu(x^\sigma) dx^\mu, |\omega^l_\mu| \neq 0)$$

except undergoing *extended* orthogonal transformations of $\frac{1}{2}(1+\delta^4_l)\omega^l$, having discovered the *extended* orthogonal transformations with functions of coordinates (x^σ) as coefficients and simplified calculations extremely by taking $\omega^l_\mu(x^\sigma)$ and $\Lambda^\lambda_{\mu\nu}$ in place of $g_{\mu\nu} = \omega^l_\mu \omega^l_\nu$ and $\{^{\lambda}_{\mu\nu}\}$ respectively, where

(4) $$\Lambda^\lambda_{\mu\nu} \overset{\text{def}}{=} \Omega^\lambda_l \frac{\partial \omega^l_\mu}{\partial x^\nu} \equiv -\omega^l_\mu \frac{\partial \Omega^\lambda_l}{\partial x^\nu}$$

is the *parameter of teleparallelism* of $\omega^l_\mu(x^\sigma)$ and $\Omega^\lambda_l(x^\sigma)$, and

(5) $$\Omega^\lambda_l \omega^l_\mu = \delta^\lambda_\mu \iff \Omega^\lambda_m \omega^l_\lambda = \delta^l_m,$$

the δ's being the Kronecker deltas. The equations of motion of a free particle were

(6) $$\frac{d^2\xi^l}{dS^2} = \frac{d}{dS}\frac{\omega^l}{dS} \equiv \omega^l_\lambda \left\{ \frac{d^2x^\lambda}{dS^2} + \Lambda^\lambda_{\mu\nu} \frac{dx^\mu}{dS}\frac{dx^\nu}{dS} \right\} = 0,$$

whose finite equations are