161. On the Gibbs Phenomenon for Quasi-Hausdorff Means

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1. The Hausdorff transformation is defined as transforming the sequence $\{s_\nu\}$ into the sequence $\{h_n\}$ by means of the equation

$$h_n=\sum_{\nu=0}^{n}\binom{n}{\nu}s_\nu\int_0^1 r^\nu(1-r)^{n-\nu}d\psi(r),$$

where the weight function $\psi(r)$ is of bounded variation in the interval $0\leq r\leq 1$. This transformation is regular if and only if

$$\psi(1)-\psi(0)=1,$$

and if $\psi(r)$ is continuous at $r=0$. We may assume that $\psi(0)=0$, then the above conditions become

$$\psi(1)=1,\quad \psi(+0)=\psi(0)=0.$$

Corresponding to any fixed number r with $0<r\leq 1$, if we put $\psi(x)=e_r(x)$, where

$$e_r(x)=\begin{cases}0 & \text{for}\quad 0\leq x<r\\ 1 & \text{for}\quad r\leq x\leq 1,\end{cases}$$

then the Hausdorff transformation reduces to the Euler transformation, i.e.

$$\sigma_n(r)=\sum_{\nu=0}^{n}\binom{n}{\nu}r^\nu(1-r)^{n-\nu}s_\nu.$$

The case $r=1$ corresponds to the ordinary convergence. For the fundamental properties of the Hausdorff and Euler transformations, see, e.g., G. H. Hardy ([1], Chapters VIII and XI).

Let $\phi(t)$ denote the function of period 2π and equal to $\frac{1}{2}(\pi-t)$ in the interval $0<t<2\pi$. Then $\phi(t)$ has a simple discontinuity at the origin: its Fourier series is

$$\sum_{n=1}^{\infty}\frac{\sin nt}{n}. \tag{1}$$

O. Szász [12, 13] investigated the Gibbs phenomenon of this series for the Hausdorff and Euler means. Here we put $s_0=s_0(t)=0$ and $s_\nu=s_\nu(t)=\sum_{n=1}^{\nu}\frac{\sin nt}{n}$. He proved the following

THEOREM 1. *For the regular Hausdorff means of* (1) *we have*

$$\lim_{n\to\infty}h_n(t_n)=\int_0^1 d\psi(r)\int_0^\tau\frac{\sin ry}{y}dy,$$

as $nt_n\to\tau$ *with* $0\leq\tau\leq+\infty$.

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