160. The Asymptotic Behaviour of the Solution of a Semi-linear Partial Differential Equation Related to an Active Pulse Transmission Line

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1. **Introduction.** J. Nagumo [1] proposed as active pulse transmission line simulating an animal nerve axon. The equation of propagation of his line is the following:

$$(1)\qquad \frac{\partial^2 u}{\partial t^2}=\frac{\partial^3 u}{\partial x^2 \partial t}-\mu(1-u+\varepsilon u^2)\frac{\partial u}{\partial t}-u \qquad \begin{matrix}\mu>0,\ \varepsilon>0\\ x>0,\ t>0\end{matrix}$$

with the boundary data;

$$(2)\qquad \begin{cases} u(x,0)=0 & (x\geq 0)\\ u_t(x,0)=0 & (x\geq 0)\\ u(0,t)=\psi(t) & (t\geq 0),\ \psi(t)\equiv 0 \quad \text{for} \quad t\geq t_0.\end{cases}$$

In this note, we consider some asymptotic behaviours of the solution for the equation of related type with the same boundary data: Our equation is the following:

$$(3)\qquad \frac{\partial^2 u}{\partial t^2}=\frac{\partial^3 u}{\partial x^2\partial t}-f'(u)\frac{\partial u}{\partial t}-g(u).$$

At first, we remark that the existence of global solutions for this problem (3) with boundary data (2) where $\psi(t)\in C^2$ is assumed was completely proved by R. Arima and Y. Hasegawa [2] under the conditions:

$$(4)\qquad \begin{cases} -K_1\leq f'(u)\leq K_0(u^2+1),\\ |g(u)|\leq K_2(u^2+|u|),\\ G(u)=\displaystyle\int_0^u\{-g(z)\}dz\leq K_3u^2,\end{cases}$$
$$g(u),\ f'(u)\in C^1.$$

Throughout this paper, we always assume that $f'(u)$, $g(u)$ satisfy this condition (4).

Our results are divided into two parts. The one is the case $g(u)\equiv u$, the other is the case $g(u)\equiv 0$. For the first case, we can prove that any solution $u(x,t)$ tends uniformly to zero, when t tends to $+\infty$, under the additional condition (5), which corresponds to the limitation $\varepsilon>\frac{3}{16}$ in (1). For the second case we can show the existence of a threshold value for the boundary data (Prop. 3) and a sort of asymptotic value under another additional conditions (Prop. 4), (9), (11), which is independent of (5).

We remark also that the summability in x, of $u(x,t)^2$ and $u_x(x,t)^2$,