158. *On Fields of Division Points of Algebraic Function Fields of One Variable*

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Let K be a field of algebraic functions of one variable over an algebraically closed constant field k. Let $D_0(K)$ be the group of all the divisors of degree 0 of K and $C(K)$ the divisor class group of K, i.e. the factor group of $D_0(K)$ by the subgroup consisting of all the divisors which are linearly equivalent to 0 (in notation: ~ 0). We use the additive notation for the group laws of $D_0(K)$ and $C(K)$. Let g be the genus of K. Then, for a natural number n prime to the characteristic of k, it is known that there exist exactly n^{2g} elements $c_1, \cdots, c_N$ ($N=n^{2g}$) of $C(K)$ such that $nc_i=0$. We call these c_i the *n-division points* of $C(K)$.

Let $D_1, \cdots, D_N$ be an *arbitrary* system of representative divisors of the classes $c_1, \cdots, c_N$ (c_i is the divisor class containing D_i). Then nD_i is linearly equivalent to 0 and so there exist N elements $x_1, \cdots, x_N$ of K such that the divisor (x_i) of x_i is equal to nD_i. We consider the subfield $K_n=k(x_1, \cdots, x_N)$ of K generated by $x_1, \cdots, x_N$ over k. We shall call such a field K_n a *field of n-division points of K*. Since there are infinitely many choices of systems of representative divisors of the classes c_i, there are also, for a fixed given n, infinitely many fields of n-division points of K. We note that if $n>1$, K_n has the transcendental degree 1 over k and so the degree $[K:K_n]$ is finite. In fact, for $n>1$, some c_i is not equal to 0 and so x_i is not a constant.

Now we shall prove the following

Theorem. *Suppose $g\geqq 2$. Let $l\geqq 3$ be a prime number different from the characteristic of k. Then, for any field K_l of l-division points of K, K is purely inseparable over K_l. In particular, if the characteristic of k is 0, we have $K=K_l$.*

The case where $l=2$ (and the characteristic $=0$) was considered by Arima in [1]. We shall prove our theorem in the separable case by the same idea.

The proof of the theorem is divided into two cases.

1) First we consider the case where K is separable over K_l. We assume that $K\neq K_l$ and deduce a contradiction. Let g_0 be the genus of K_l. Then, as $g\geqq 2$ and $K\neq K_l$, we have $g>g_0$ by the formula of Hurwitz. We denote by $(x_i)_K$ and $(x_i)_{K_l}$ the divisors of the function