157. *On the Immersibility of almost Parallelizable Manifolds*

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1. M. W. Hirsch has shown that an almost parallelizable n-manifold is immersible in the Euclidean $(n+k)$-space R^{n+k} if $n<2k$ [1]. He has proved the result by making use of a result due to M. Kervaire that the Smale invariant of an immersion of n-sphere in R^{n+k} vanishes if $n\leqq 2k-2$ [4]. In this paper we shall prove the following*)

Proposition 1. An almost parallelizable n-manifold is immersible in R^{n+1} if $n\not\equiv 0 \pmod 4$.

Proposition 2. If $n\equiv 0 \pmod 4$, an almost parallelizable n-manifold is in general not immersible in R^{n+1}. In particular, Hirsch's result is best possible for $n=4$. The authors wish to express their thanks to prof. K. Aoki and Prof. T. Kaneko for their many valuable suggestions and several discussions.

2. In the following discussions, all manifolds are considered as connected, orientable C^∞ manifolds. By immersion $f: M^n\to R^p$ we mean C^∞ map whose Jacobian matrix has rank $n=\dim M^n$ at each point of M. A homeomorphic immersion will be called imbedding. A manifold M^n will be called parallelizable if its tangent bundle is trivial, we say M^n is almost parallelizable if M^n-x is parallelizable for some $x\in M^n$. M^n will be called π-manifold if M^n is imbedded in R^{n+k} $(k>n)$ with trivial normal bundle ν.

Since, a non-closed (i.e. non-compact or with boundary) almost parallelizable manifold is parallelizable, hence it is immersible in R^{n+1} (Theorem 6.3 of [2]). Therefore we may consider only closed manifolds.

Let o_n denote the obstruction to the extension over M^n of the cross section over M^n-x; o_n is an element of $\pi_{n-1}(SO(k))$. Now let

$$J: \pi_{n-1}(SO(k))\to\pi_{n+k-1}(S^k)$$

be the Hopf-Whitehead homomorphism, it is well known that $J(o_n)=0$. Moreover the result of J. F. Adams [6] implies that homomorphism J is injective for $\pi_{n-1}(SO(k))=Z_2$ $(k>n)$.

In the case of $n\not\equiv 0 \pmod 4$, $\pi_{n-1}(SO(k))=0$ or Z_2 according to whether $n\equiv 3, 5, 6, 7 \pmod 8$ or $n\equiv 1, 2 \pmod 8$ respectively. From this it follows that in the case of $n\equiv 3, 5, 6, 7 \pmod 8$, $o_n=0$. In the case $n\equiv 1, 2 \pmod 8$, o_n is also zero, since it belongs to the kernel

*) Details will appear in [7].