6. On Some Topologies in the Universal Hilbert Space

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1. It is one of the most important problem of the quantum field theory that what kind of Hilbert (or linear) space we must construct corresponding to the set of all physical states, i.e., in what kind of space we must consider representation of the operator algebra of physical observables.

As many authors [1–3] pointed out, the inequivalency of irreducible representations of cannonical commutation relation (or of similar operator algebra [4–7], causes the different types of field theory. In addition to the inequivalency, the problem of orthogonality is another serious problem.

O. Miyatake [9, 10], and Van Hove [2] pointed out the problem of the orthogonality as follows. Let H_0 be the free Hamiltonian of neutral scaler meson field interacting with the fixed point source. Let H_1 be the total Hamiltonian of the coupled system. Then the Hilbert space $\boldsymbol{H}_1$ spanned by all eigenvectors of H_1 is perpendicular to the Hilbert space $\boldsymbol{H}_0$ spanned by all eigenvectors of H_0. This orthogonality deny the applicability of customary perturbation in which a state of $\boldsymbol{H}_1$ is expanded by the complete orthonormal system of $\boldsymbol{H}_0$.

Further we may well imagine that not only in the fixed source model but also in the real field similar orthogonality relation would hold, and we can imagine also well that the customary perturbative methods would break down in the real field. K. O. Friedrichs referred these problem [1] and proposed that both spaces should be considered as subspaces of a universal Hilbert space introduced by J. Von Neumann [11].

In this article we consider about universality of this space (§ 2) and the relation to the customary occupation number representation (§ 3) and investigate the new topology to obtain the new perturbative method between orthogonal spaces (§ 4–6).

2. In the quantum field theory one frequently constructs the Hilbert spaces of state vectors in the following several ways.

(1) One assumes Hamiltonian of the system, and finds the eigenvectors of Hamiltonian, then takes closure of the linear aggregate of the eigenvectors (e.g. [2, 9, 10]).

(2) One assumes the algebraic relations of operators which