3. *Certain Embedding Problems of Semigroups*

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1. By a *left translation* of a semigroup S we mean a transformation λ of $S, x \rightarrow x\lambda$, satisfying $(xy)\lambda=(x\lambda)y$, for all x, y in S. A *right translation* of S is a transformation ρ satisfying $(xy)\rho=x(y\rho)$, for all x, y in S. A left translation λ and a right translation ρ are said to be *linked* if $x(y\lambda)=(x\rho)y$, for all x, y in S. We note that for each a in S, the transformation λ_a defined by $x\lambda_a=ax$, for all x in S, is a left translation of S, the transformation ρ_a defined by $x\rho_a=xa$, for all x in S, is a right translation of S, and λ_a and ρ_a are linked. We call λ_a an *inner left translation* of S, ρ_a an *inner right translation* of S. A semigroup S is said to be *weakly reductive* if, for any a, b in S, $ax=bx$ and $xa=xb$, for all x in S, imply $a=b$.

It was proved in [1] that a weakly reductive semigroup S can be embedded into a semigroup T so that

(1) S is an ideal of T,

(2) every left translation of S is induced by some inner left translation of T, and every right translation of S is induced by some inner right translation of T,

if and only if

(3) every left translation of S is linked with some right translation of S, and *vice versa.*

However, the general case in which weak reductivity is not assumed was open. In this paper we shall give necessary and sufficient conditions for an arbitrary semigroup S so that it can be embedded into a semigroup T with the properties (1) and (2). The special case for weakly reductive semigroups will follow as a corollary. We shall also discuss the embedding of a semigroup S into a semigroup T under conditions somewhat weaker than (1) and (2).

2. The open problem in [1] can be solved as follows:

Theorem 1. A semigroup S can be embedded into a semigroup T so that

(1) $ST \subseteq S$, $TS \subseteq S$,

(2) for every left translation λ of S there exists a in T such that $x\lambda=ax$, for all x in S, and for every right translation ρ of S there exists b in T such that $x\rho=xb$, for all x in S,

if and only if

(3) every left translation of S is linked with some right translation of S, and *vice versa,*