22. Construction of Finite Commutative z-Semigroups

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(Comm. by Kinjirô KUNUGI, M.J.A., Feb. 12, 1964)

§ 1. **Introduction.** As defined by Tamura [4], a semigroup is called a *z-semigroup* if it has a zero element, 0, but has no idempotent except 0. In particular, for a finite commutative semigroup S it is easily seen that S is a z-semigroup if and only if it satisfies the following two conditions:

(1) S has a zero element 0

and (2) $S\supsetneqq S^2\supsetneqq\cdots\supsetneqq S^p=\{0\}$ for some positive integer p.[1]

If $S\backslash S^2$ is non-empty, every element of $S\backslash S^2$ is called a *prime element* of S.

In the case of $p=1$ or $p=2$, S satisfies the following

(3) $S=\{0\}$

or (4) $xy=0$ for any $x, y\in S$,

respectively.

Such a semigroup S is called a *trivial z-semigroup* or a *null semigroup*, corresponding to $p=1$ or $p=2$.

Now, the problem of construction of finite commutative z-semigroups occupies an important part in the problem of construction of finite commutative semigroups. In this paper, we shall deal with this problem and present a method of constructing all possible commutative z-semigroups of a given order. The proofs are omitted and will be given in detail elsewhere.[2]

§ 2. **Commutative z-semigroups of order n.** At first, we have

Theorem 1. *For any positive integer n, there exists a commutative z-semigroup of order n.*

Let G be a semigroup with a zero element 0. The subset A of G, where $A=\{x: x\in G, xy=yx=0 \text{ for all } y\in G\}$, is a subsemigroup of G. We shall call A the *annihilator* of G.

Lemma 1. *The annihilator of a non-trivial, finite commutative z-semigroup has a non-zero element (see also Tamura [3]).*

Lemma 2. *Let S be a commutative z-semigroup of order $n+1$ $(n\geqq 1)$. Let 0 be the zero element of S and let u be a non-zero element contained in the annihilator of S. Then the set $\{0, u\}$ is both a null subsemigroup and an ideal of S, and the factor semigroup $D=S/\{0, u\}$ of S mod $\{0, u\}$ in the sense of Rees [2] is a commutative z-semi-*

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1) $A\supsetneqq B$ means 'B is a proper subset of A'.

2) This is an abstract of a paper which will appear elsewhere.