19. On Postulate-Sets for Newman Algebra and Boolean Algebra. I

By Yuki WOOYENAKA

Japan Women's University, Tokyo

(Comm. by Kinjirô KUNUGI, M.J.A., Feb. 12, 1964)

Introduction. There are many sets of postulates for Boolean algebra given by various scholars [1]. Moreover, M. H. Stone has shown among other things, that one can subsume the theory of Boolean algebra under the theory of Boolean ring [2]. The sets of postulates for Boolean ring (or generalized Boolean algebra) were given by Stabler [3] and Bernstein [4]. On the other hand, Newman has given the most remarkable system known as Newman algebra [5, 6] including both Boolean algebra and Boolean ring.

We shall give in this paper two kinds of postulate-sets for Newman algebra as Set I and Set II. The idea of the postulates of Set I was suggested to me by Bernstein's dual-symmetric definition of Boolean algebra [7] where the distributive law $a\vee(bc)=(a\vee b)(a\vee c)$ is eliminated. In Set II, we have replaced the commutative laws for addition and multiplication by axioms B_1', C', and C_1' below. This set has not an exactly dual, but a nearly dual form. And this set has a form quite close to that of the postulates of Newman algebra due to Birkhoff: our axioms B_1, B_1' are just the same as the axioms N1, N1$'$ of Birkhoff [6: p. 155], our C_1', C_1 are nearly like N2, N3, and our E' corresponds to N4.

The paper consists of three paragraphs. The first gives the postulates of Set I and Set II and shows that each set is equivalent with the system of Newman algebra [6]. Four kinds of postulate-sets for Boolean algebras, Set I_1, Set I_2^*, Set II_1, and Set II_2^* will be derived from Set I and Set II respectively by Newman's decomposition theorem. The second deals with the construction of some independence-systems with eight elements. In constructing these systems we give several theorems where we shall see how helpful Stone's theory of Boolean ring [2] will also be for our purpose. The third gives the independence proofs for the four new sets for Boolean algebras.

In concluding, we should like to give the following remark. G. D. Birkhoff and G. Birkhoff say in the introduction of their paper, that they have made Newman's argument such shorter and simpler in adding a dependent postulate $0+a=a$. Our Sets I and II are independent sets and fit for Birkhoff's argument. As such our sets may be regarded as one of the suitable systems to characterize