43. On the Lebesgue Constants for Quasi-Hausdorff Methods of Summability. II

By Kazuo ISHIGURO

Department of Mathematics, Hokkaido University, Sapporo

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§ 5. For the proof of Theorem 1, we shall prove the following **Lemma.**

$$L_\delta^*(n;\psi)=\frac{2}{\pi}\int_1^{\sqrt{n}}\frac{du}{u}\left|\int_\delta^1 \sin\frac{u}{r}\,d\psi(r)\right|+ \tag{5.1}$$
$$+\frac{2}{\pi^2}\,|\psi(1)-\psi(1-0)|\log n+o(\log n).$$

It may be noted that the upper limits of the Stieltjes integrals in (3.4) and (5.1) are different.

Proof. We shall use the method of L. Lorch and D. J. Newman [5]. In order to simplify the following calculations, we shall prove

$$L_\delta^*(n-1;\psi)=\frac{2}{\pi}\int_1^{\sqrt{n}}\frac{du}{u}\left|\int_\delta^1 \sin\frac{u}{r}\,d\psi(r)\right|+ \tag{5.2}$$
$$+\frac{2}{\pi^2}\,|\psi(1)-\psi(1-0)|\log n+o(\log n).$$

It is easily seen that (5.1) and (5.2) are equivalent for large n.

Replacing the factor $\{\sin(2n+1)u\}/\sin u$ by $\{\sin 2(n+1)u\}/u$ in (3.4) induces a bounded error, we obtain, from (2.2),

$$L_\delta^*(n-1;\psi)=\frac{2}{\pi}\int_0^{\pi/2}|K_n(u)|\frac{du}{u}+O(1),$$

where

$$K_n(u)=\int_\delta^1\left(\frac{1}{1+\dfrac{4(1-r)}{r^2}\sin^2 u}\right)^{\frac{n}{2}}\sin\frac{2nu}{r}\,d\psi(r). \tag{5.3}$$

For fixed ε and A with $0<\varepsilon<1<A$, we put

$$\int_0^{\pi/2}|K_n(u)|\frac{du}{u}=\int_0^{\frac{\varepsilon}{\sqrt{n}}\delta^*}+\int_{\frac{\varepsilon}{\sqrt{n}}\delta^*}^{\frac{A}{\sqrt{n}}\delta^*}+\int_{\frac{A}{\sqrt{n}}\delta^*}^{\pi/2}=I_1+I_2+I_3,$$

where $\delta^*=\delta/\sqrt{2(1-\delta)}$.

As to I_1: In the interval $0\le u\le\frac{\varepsilon}{\sqrt{n}}\delta^*$, we have

$$1\ge\left(\frac{1}{1+\dfrac{4(1-r)}{r^2}\sin^2 u}\right)^{\frac{n}{2}}\ge 1-\varepsilon^2,$$