42. On the Lebesgue Constants for Quasi-Hausdorff Methods of Summability. I

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§ 1. The quasi-Hausdorff transformation (H^*, ψ) is defined as transforming the sequence $\{s_\nu\}$ into the sequence $\{h_n^*\}$ by means of the equation

$$h_n^* = \sum_{\nu=n}^{\infty} \binom{\nu}{n} s_\nu \int_0^1 r^{n+1}(1-r)^{\nu-n} d\psi(r),$$

where the weight function $\psi(r)$ is of bounded variation in the interval $0 \leq r \leq 1$. This transformation is regular if and only if

$$\psi(1) - \psi(+0) = 1.$$

We may assume, in the following, that

$$\psi(1) = 1, \quad \psi(+0) = 0.$$

Corresponding to any fixed number r with $0 < r \leq 1$, if we put $\psi(x) = e_r(x)$, where

$$e_r(x) = \begin{cases} 0 & \text{for } 0 \leq x < r \\ 1 & \text{for } r \leq x \leq 1, \end{cases}$$

then the quasi-Hausdorff transformation reduces to the circle transformation (γ, r).

The Lebesgue constant of order n for the method (H^*, ψ) is then defined to be

(1.1) $$L^*(n; \psi) = \frac{2}{\pi} \int_0^\pi dt \left| \sum_{\nu=n}^{\infty} \binom{\nu}{n} \frac{\sin(\nu+\frac{1}{2})t}{2 \sin \frac{1}{2}t} \int_0^1 r^{n+1}(1-r)^{\nu-n} d\psi(r) \right|.$$

As is well known, if $L^*(n; \psi) \to \infty$ as $n \to \infty$, then there is a continuous function whose Fourier series is not summable (H^*, ψ) for at least one point.

The Lebesgue constants for the method (γ, r) were studied by L. Lorch [4] and by the author [2]. On the other hand, first A. E. Livingston [3] and recently L. Lorch and D. J. Newman [4] studied the Lebesgue constants for the regular Hausdorff methods of summability in detail. For the definition and the properties of the Hausdorff methods, see, e.g., G. H. Hardy [1]. We shall study, in this note, the Lebesgue constants for the quasi-Hausdorff methods of summability.

§ 2. From (1.1), we get