41. *Extension of a Theorem of Hyslop on Absolute Cesàro Summability*

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1. *Definitions and notations.* We shall denote the nth Cesàro-sum, Cesàro-mean and Cesàro-transformed term of order κ $(\kappa>-1)$ of the series $\sum a_n$ by S_n^κ, s_n^κ and a_n^κ respectively, and the corresponding sum, mean and term for the series $\sum \lambda_n a_n$ by $S_{n,\lambda}^\kappa$, $s_{n,\lambda}^\kappa$ and $a_{n,\lambda}^\kappa$ respectively.

Thus
$$s_n^\kappa=\frac{S_n^\kappa}{A_n^\kappa}=\frac{1}{A_n^\kappa}\sum_{\nu=0}^{n} A_{n-\nu}^{\kappa-1}s_\nu=\frac{1}{A_n^\kappa}\sum_{\nu=0}^{n} A_{n-\nu}^{\kappa}a_\nu,$$
where A_n^κ is defined by the identity
$$(1-x)^{-\kappa-1}=\sum A_n^\kappa x^n \qquad (|x|<1),$$
and
$$a_n^\kappa=s_n^\kappa-s_{n-1}^\kappa.$$
Similarly,
$$s_{n,\lambda}^\kappa=\frac{S_{n,\lambda}^\kappa}{A_n^\kappa}=\frac{1}{A_n^\kappa}\sum_{\nu=0}^{n} A_{n-\nu}^{\kappa}\lambda_\nu a_\nu$$
and
$$a_{n,\lambda}^\kappa=s_{n,\lambda}^\kappa-s_{n-1,\lambda}^\kappa.$$

A series is said to be absolutely summable (C,κ), or summable $|C,\kappa|$, $\kappa>-1$, if
$$\sum|a_n^\kappa|=\sum|s_n^\kappa-s_{n-1}^\kappa|<\infty.$$

We observe that, by a well-known identity, due to Kogbetliantz,*)
$$a_n^\kappa=s_n^\kappa-s_{n-1}^\kappa=n^{-1}t_n^\kappa=n^{-1}(A_n^\kappa)^{-1}T_n^\kappa,$$
where t_n^κ and T_n^κ are the nth Cesàro-mean and sum of order κ of the sequence $\{na_n\}$. Thus the summability $|C,\kappa|$ of $\sum a_n$ is the same as the convergence of the series $\sum n^{-1}|t_n^\kappa|$, or $\sum n^{-1}(A_n^\kappa)^{-1}\cdot|T_n^\kappa|$.

Moreover, since
$$T_n^\kappa=nA_n^\kappa a_n^\kappa,$$
and
$$A_n^\kappa\sim\frac{n^\kappa}{\Gamma(\kappa+1)},\quad \text{for } \kappa\neq-1,-2,-3,\cdots,$$
we see that the summability $|C,\kappa|$ of $\sum a_n$ is the same as the convergence of
$$\sum n^{-(\kappa+1)}|na_n^\kappa A_n^\kappa|.$$
Similarly, according to our notations, the summability $|C,p|$ of

*) Kogbetliantz [3], [4].