38. The Mean Continuous Perron Integral

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1. Introduction. H. W. Ellis [2] has introduced the GM-integral descriptively whose indefinite integral is mean continuous. The GM-integral is an extension of the CP-integral defined by J. C. Burkill [1]. The aim of this paper is to define an integral of the Perron type which is equivalent to the GM-integral. We call this integral the mean continuous Perron integral or MP-integral.

In § 2 we shall define the MP-integral and prove its fundamental properties. The equivalence between the GM-integral and the MP-integral will be considered in § 3. The proof is essentially based on the method used by J. Ridder ([4], pp. 7–8).

2. The mean continuous Perron integral.

Definition 2.1 ([2], p. 114). If $f(x)$ is general Denjoy integrable on $[a, b]$ then we write

$$M(f, a, b)=\frac{1}{b-a}\int_a^b f(t)dt.$$

If $\lim_{h\to 0} M(f, c, c+h)=f(c)$ then $f(x)$ is termed mean continuous or M-continuous at c.

Definition 2.2. A finite function $f(x)$ is said to be $\underline{AC}$ on a set E if to each positive number ε, there exists a number $\delta>0$ such that

$$\Sigma\{f(b_k)-f(a_k)\}>-\varepsilon$$

for all finite non-overlapping sequence of intervals $\{(a_k, b_k)\}$ with end points on E and such that $\Sigma(b_k-a_k)<\delta$. There is a corresponding definition of $\overline{AC}$ on E. If the set E is the sum of a countable number of sets E_k on each of which $f(x)$ is $\underline{AC}$ then $f(x)$ is termed $\underline{ACG}$ on E. Similarly we can define $\overline{ACG}$ on E. If $f(x)$ is both $\underline{ACG}$ and $\overline{ACG}$ on E then we say that $f(x)$ is ACG on E.

Definition 2.3 ([2], p. 115). A finite function $f(x)$ is said to be $(\underline{ACG})$ on E if E is the sum of a countable number of closed sets E_k on each of which $f(x)$ is $\underline{AC}$. If "$\underline{AC}$" is replaced by "$\overline{AC}$", then the corresponding definition of $(\overline{ACG})$ is obtained. If $f(x)$ is both $(\underline{ACG})$ and $(\overline{ACG})$ on E then $f(x)$ is termed (ACG) on E.

Definition 2.4. Let $f(x)$ be defined on an interval $[a, b]$. The function $U(x)$ $[L(x)]$ is called upper [lower] function of $f(x)$ in $[a, b]$ if