31. *A Local Asymptotic Law for the Transient Stable Process*

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In a preceding paper [5], the author has obtained a criterion of an upper class and a lower class concerning the asymptotic behaviours of the transient symmetric stable process when the time parameter t tends to infinity. The concept of the upper class and the lower class with respect to certain stochastic processes can be defined also for a neighbourhood of $t=0$ as for the case of $t=\infty$. In fact, S. Watanabe and the author, in [6], have given two criteria with respect to Cauchy process on a line. For the Brownian motion it is well-known that, by virtue of the principle of projective invariance of P. Lévy [4], the criterion for the former can be derived from that for the latter and *vice versa*. Unfortunately such a principle is unknown for stable process and it does not seem that the generalization of Lévy proof for such a process is possible. So in this paper we shall give the criterion for local case of the transient stable process directly.

There are various generalization of the "infinite-sum" part of the Borel-Cantelli lemmas for dependent events. Among them, that of Chung-Erdös [1] is the most celebrated. Recently J. Lamperti [3] obtained a very simple lemma and essentially the same result is also indicated by Ciesielski-Taylor [2]. However, their lemmas are not delicate enough to deal with our problem. Suggested by their results, we show in Lemma B the following fact that the condition (ii) of Chung-Erdös [1] can be removed if we assume that 0–1 law is valid.

Let $\{X(t, w);\ 0\leqq t<\infty\}$ be the symmetric stable process of index α in R^N. We concern only with the transient case, that is, the case $\alpha<N$. As usual we assume $X(0, w)=0$ with probability one. For any positive monotone non-increasing function $g(t)$ defined for large t's, we put

$$F(w)=\left\{t;\ |X(t, w)|\leqq t^{1/\alpha}g\left(\frac{1}{t}\right)\right\}$$

and if

$$P\{w;\ \inf_{t\in F(w)} t>0\}=0 \qquad (\text{or } =1) \tag{1}$$

then we say that $g(t)$ belongs to the upper class $\mathfrak{U}_0$ (or the lower class $\mathfrak{L}_0$) with respect to $X(t)$. Then we show the following