62. *Use of the Function* $\sin x/x$ *in Gravity Problems*

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The paper of Y. Tomoda and K. Aki,[1] under the above title, describes a method of such simplicity and ease of application as to make it worthy of any small further clarification. Let us look at the "convergence" of the series used. Quotes are used because observed values enter the series as well as analytical expressions, so convergence in the ordinary strict sense may not be applicable.

In their paper, Tomoda and Aki, for clear exposition, take grid points at $\pm n\pi$ with corresponding gravity anomaly values $\Delta g_{\pm n}$ and project downwards to a depth, d in radians. From this downward projection of Δg, a corresponding surface mass density is found, which will yield the same anomaly field as the original one at the datum surface. Then they pass to the more useful case where ξ and δ are the actual horizontal and vertical distances measured in any convenient units and a is the grid spacing between gravity stations measured in the same units. We start from this latter stage of their work.

Let the ratio δ/a be r. Then the mass surface density under the i^{th} grid point will be

$$M(i, r)=\frac{1}{2\pi k^2}\sum_{j=-\infty}^{+\infty}\varphi_j(r)\Delta g_{i+j}$$
$$=\frac{\varphi_0(r)\Delta g_i}{2\pi k^2}+\frac{1}{2\pi k^2}\sum_{j=1}^{\infty}\varphi_j(r)(\Delta g_{i+j}+\Delta g_{i-j}) \qquad (1)$$

where (see reference 1), p. 446) k^2 is the gravitational constant and

$$\varphi_j(r)=r\{(-1)^j e^{\pi r}-1\}/\{\pi(j^2+r^2)\}=(-1)^j e^{\pi r}\psi_j(r)-\psi_j(r)$$
$$=\theta_j(r)-\psi_j(r) \qquad (2)$$

with

$$\psi_j(r)=r/\{\pi(j^2+r^2)\}. \qquad (3)$$

The terms $\psi_i(r)\to r/\pi j^2$ as j increases, for practical values of r, such as $r=1/2$. $\theta_j(r)$ is an alternating sequence of terms of decreasing numerical values.

L. B. W. Jolley[2] (see pp. 22–23) lists summation formulae giving

$$\sum_{j=1}^{\infty}\psi_j(r)=(1/2)\coth\pi r-1/(2\pi r),$$
$$\sum_{j=1}^{\infty}\theta_j(r)=(1/2)(e^{\pi r}/\sinh\pi r)-e^{\pi r}/(2\pi r). \qquad (4)$$

So if we use the constant lateral extensions of the Δg values, as Tomoda and Aki[1] (p. 446 bottom lines) do for Vening Meinesz's gravity profile #17 in the East Indies, then we can compute $M(i, r)$