# 60. On the Covering Dimension of Product Spaces

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Let $X$ and $Y$ be normal spaces. As for the covering dimension of the product space $X\times Y$ we have known several cases for which the following relation

(A) $$\dim(X\times Y)\leqq \dim X+\dim Y$$

holds.

Especially when $Y$ is a separable metrizable space, (A) has been proved in each of the following cases.

(a) $X$ is metrizable ([2]).

(b) $X$ is countably paracompact and normal, and $Y$ is locally compact ([2]).

In the present paper we shall prove (A) under the conditions that $Y$ is separable metrizable and $X\times Y$ is countably paracompact and normal.

Recently E. Michael [1] has given a non-normal space $X\times Y$ which is a product space of a hereditarily paracompact normal space $X$ with a separable metric space $Y$. This space $X\times Y$ is not 0-dimensional, nevertheless $X$ and $Y$ are 0-dimensional; thus (A) does not hold.

Accordingly the normality of $X\times Y$ is indispensable.

The idea of the proof for our theorem is based on the "basic coverings" introduced by K. Morita ([3]).

**1.** Henceforth $Y$ always means a separable metrizable space.

**Lemma 1.** *Suppose that $\dim Y=n$ and let $s$ be an arbitrary positive integer: then there are locally finite countable coverings*

$$\mathfrak{B}_i^{(l)}=\{V_{i\alpha}^{(l)}\mid \alpha=1,2,\cdots\}\ (1\leqq l\leqq s;\ i=1,2,\cdots)$$

*satisfying the following conditions* (i) *and* (ii).

(i) $\bigcup_i \mathfrak{B}_i^{(l)}$ *is an open basis of $Y$ for any $l\,(1\leqq l\leqq s)$.*

(ii) *The order of the family $\{\mathfrak{B}V_{i\alpha}^{(l)}\mid i,\alpha=1,2,\cdots;1\leqq l\leqq s\}$ is at most $n$.* (Here $\mathfrak{B}V_{i\alpha}^{(l)}$ means $\overline{V_{i\alpha}^{(l)}}-V_{i\alpha}^{(l)}$.)

*Proof.* The existence of $\mathfrak{B}_i^{(l)}$ satisfying (i) is well known (e.g. [3]), and these may be considered as countable coverings for any $i$ and $l$, according to separability of $Y$. Moreover, the existence of such $\mathfrak{B}_i^{(l)}$ that satisfy (ii) is assured by the shrinkability of the covering $\mathfrak{B}_i^{(l)}$ and [4].

Put $$W^{(l)}(\alpha_1,\alpha_2,\cdots,\alpha_i)=V_{1\alpha_1}^{(l)}\cap V_{2\alpha_2}^{(l)}\cap\cdots\cap V_{i\alpha_i}^{(l)}.$$