58. On the Uniqueness of the Cauchy Problem for Semi-elliptic Partial Differential Equations. III

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1. **Introduction.** In this note we shall remark the superfluity of the condition IV of the uniqueness theorems obtained in the previous note [5]. As Theorem 1 is fundamental among Theorems in [5], we shall only indicate the modifications to be done in its proof. That theorem is related as the following:

Theorem 1 in [5]. $P(x, D)=P_0(x, D)+Q(x, D)$,

$$P_0(x, D)=\sum_{|\alpha:m|=1} a_\alpha(x)D^\alpha,\ Q(x, D)=\sum_{j=1}^{n}\sum_{|\alpha:m|\leq 1-\frac{1}{m_j}} a_\alpha(x)D^\alpha.^{*)}$$

I. (1) $m_1\geq m_j$. (2) *The coefficients of* $P_0(x, D)$ *are in* $C^{2|m|}(\Omega)$ *and those of* $Q(x, D)$ *are in* $C(\Omega)$ *and bounded on* $\bar{\Omega}$, *where* Ω *is a domain containing* $x=0$. (3) *For* $\alpha=(m_1, 0,\cdots, 0)$, $a_\alpha(0)\neq 0$.

II. $P_0(x, D)$ *is semi-elliptic at* $x=0$, *i.e.* $P_0(0, \xi)$ *does not vanish for any non-zero real vector* ξ.

III. *Let* $\zeta_1=\zeta_1(\tilde{\xi})$ *be a root of* $P_0(0, \zeta_1, \tilde{\xi})=0$, *then* $P_0^{(1)}(0, \zeta_1, \tilde{\xi})$ *does not vanish for any non-zero real vector* $\tilde{\xi}$.

IV. Let be $N^0=(-1, 0,\cdots, 0)$, $N=(N_1, N_2,\cdots, N_n)$ where N_j's are real, and $\xi+i\tau N=(\xi_1+i\tau N_1,\cdots, \xi_n+i\tau N_n)$ where τ is a real number. For $m_1\geq 2$ there are neighborhoods $U_0(0)$ of $x=0$, $V_0(N^0)$ of N^0, and a constant C_0 such that

$$\sum_{j=1}^{n}\sum_{|\alpha:m|=1-\frac{1}{m_j}} |(\xi+i\tau N)^\alpha|^2\leq C_0\Big[\sum_{j=1}^{n}|P_0^{(j)}(x, \xi+i\tau N)|^2+1\Big] \tag{1.1}$$

holds for any $x\in U_0(0)$, any $N\in V_0(N)$ and any $(\xi, \tau)\in \Xi^n\times R^1$, $\tau\geq 1$.

Suppose that I, II, III *and* IV *hold. Then there exist the constants* C, $\delta_0>0$, $M\geq 1$, *and for any real number* τ, δ *satisfying* $\delta<\delta_0$, $\tau\delta>M$,

$$\sum_{|\alpha:m|\leq 1}[(1+\tau\delta^2)\tau]^{m_0\left(1-\frac{1}{m_1}-|\alpha:m|\right)}\tau\int|D^\alpha u|^2\exp(2\tau\varphi_\delta(x))\,dx \tag{1.2}$$
$$\leq C\int|P(x, D)u|^2\exp(2\tau\varphi_\delta(x))\,dx$$

holds if $u\in C_0^\infty(U_\delta(0))$, *where* $\varphi_\delta(x)$ *is* $(x_1-\delta)^2+\delta\sum_{j=2}^{n}x_j^2$ *and* $U_\delta(0)$ *is a neighborhood depending on* δ.

2. **The superfluity of the condition IV.** We first used the

*) $\alpha=(\alpha_1, \alpha_2,\cdots, \alpha_n)\alpha_j$; integer ≥ 0, $m=(m_1, m_2,\cdots, m_n)m_j$; integer >0, $|\alpha:m|=\sum_{j=1}^{n}\frac{\alpha_j}{m_j}$. For the other notations, see [5].