57 *Representation of the State Vectors by Gelfand's Construction*

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§1. **Introduction.** In the previous paper [1], we have pointed out that Wightman's method using Gelfand's construction can treat only the free field.

Here, at the first step, the definition of the extended exponential function is given. (In another paper, we will give the definition of the extended exponential function which is more general and more faithful to the local field theory.) Using this definition, let's show the following facts:

(1) If the testing function's space in the expresssion $\exp(i\varphi(f))$ or $\exp(i\pi(f))$ is $(\mathfrak{S})$, the extended exponential function is same as the ordinary exponential function and we can construct only the eigenvectors of free Hamiltonian by the above construction.

Namely, using this cut-off of momenta $\boldsymbol{k}$, the extended exponential function can be reduced to the ordinary exponential function.

(2) If we wish to construct the eigenvectors of total Hamiltonian related to the field with interaction, at least the testing function δ as the element of the sequence space must be used. Namely, furthermore, the conditional convergence must be used [2].

§2. **Notations and definitions.** As the first step, the explicite form of the field functions $\varphi(\boldsymbol{x})$, $\pi(\boldsymbol{x})$, creation and annihilation operators $a^+(\boldsymbol{k})$, $a(\boldsymbol{k})$ and state vectors will be written down.

$$\varphi(\boldsymbol{x})=(1/(2\pi)^{3/2})\left\{\int a^+(\boldsymbol{k})\,e^{i\boldsymbol{kx}}d\boldsymbol{k}+\int a(\boldsymbol{k})\,e^{-i\boldsymbol{kx}}d\boldsymbol{k}\right\} \tag{1}$$

$$\pi(\boldsymbol{x})=(1/(2\pi)^{3/2})\left\{-\int ik_0a^+(\boldsymbol{k})\,e^{i\boldsymbol{kx}}d\boldsymbol{k}+\int ik_0a(\boldsymbol{k})\,e^{-i\boldsymbol{kx}}d\boldsymbol{k}\right\} \tag{2}$$

where $k_0=\sqrt{k_1^2+k_2^2+k_3^2+m^2}$.

Let's enclose the system in a box of finite volume V.

Considering the periodical extension of the above system, these formulas vary to the following formulas:

$$\varphi_V(\boldsymbol{x})=(1/\sqrt{V})[\textstyle\sum_{\boldsymbol{k}=(k_1,k_2,k_3)}(a^+(\boldsymbol{k})\,e^{i\boldsymbol{kx}}+a(\boldsymbol{k})\,e^{-i\boldsymbol{kx}})] \tag{3}$$

$$\pi_V(\boldsymbol{x})=(1/\sqrt{V})[\textstyle\sum_{\boldsymbol{k}=(k_1,k_2,k_3)}(-ik_0a^+(\boldsymbol{k})\,e^{i\boldsymbol{kx}}+ik_0a(\boldsymbol{k})\,e^{-i\boldsymbol{kx}})], \tag{4}$$

where $k_0=\sqrt{k_1^2+k_2^2+k_3^2+m^2}$, and k_1, k_2, k_3 are non-negative integers.

Hereafter, we use the following abbriviations: $a^+(\boldsymbol{k})\,e^{i\boldsymbol{kx}}\equiv a^+(\boldsymbol{k},\boldsymbol{x})$, $a(\boldsymbol{k})\,e^{-i\boldsymbol{kx}}\equiv a(\boldsymbol{k},\boldsymbol{x})$.

Creation and annihilation operators $a^+(\boldsymbol{k})$, $a(\boldsymbol{k})$ satisfy the conditions $[a^+(\boldsymbol{k}), a^+(\boldsymbol{k}')]=[a(\boldsymbol{k}), a(\boldsymbol{k}')]=0$ and $[a(\boldsymbol{k}), a^+(\boldsymbol{k}')]=\delta_{\boldsymbol{k},\boldsymbol{k}'}$.