55. On E. Lindelöf's Theorem on the Meromorphic Function of Bounded Characteristic in the Unit Circle

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1. **Introduction.** E. Lindelöf's theorem on asymptotic values is not always true for the meromorphic function of bounded characteristic. Indeed, putting $f(z)=(1-z)\exp\{(1+z)/(1-z)\}$, $f(z)$ is regular and of bounded characteristic in $|z|<1$, because $f(z)$ is the quotient of two bounded regular functions $(1-z)$ and $\exp\{-(1+z)/(1-z)\}$. Then we have easily

$$\lim_{\theta\to\pm 0} f(e^{i\theta})=0 \quad\text{and}\quad \lim_{r\to 1-0} f(r)=\infty,$$

which shows that E. Lindelöf's theorem is not true for $f(z)$.

The object of this note is to give the decisive answer to the question in what form E. Lindelöf's theorem should be modified in the case of the meromorphic function with bounded characteristic in $|z|<1$.

2. **Theorem 1.** Let $f(z)$ be the meromorphic function of bounded characteristic in the unit disk D, P a point on the unit circle C, and A a Jordan arc contained in $D\smile C$ and terminating at P. We denote by $D(A_1, A_2, \varepsilon)$ the domain bounded by the periphery of $U(P, \varepsilon)$*) and two Jordan arcs A_1, A_2 having no common point except for P.

Our main theorem is

Theorem 1. *Let $f(z)$ be meromorphic and of bounded characteristic in D. If $f(z)$ tends to a_i as $z\to P$ along A_i $(i=1, 2)$, then following alternatives are possible:*

(1) *$a_1=a_2$ and $f(z)$ tends uniformly to $a_1=a_2$ as $z\to P$ in $\overline{D}(A_1, A_2, \varepsilon)$,*

or

(2) *Picard's exceptional value in $D(A_1, A_2, \varepsilon)$ distinct from a_i $(i=1, 2)$ is at most one.*

Remark 1. Applying the theorem of Iversen-Gross ([3] p. 24) to $D(A_1, A_2, \varepsilon)$, we can conclude that following alternatives are possible:

(1) $a_1=a_2$ and the cluster set at P reduces to this single point,

or

(2) every value distinct from a_i $(i=1, 2)$, *except for at most two values,* is taken infinitely often by $f(z)$ in $D(A_1, A_2, \varepsilon)$.

Hence, Theorem 1 means that the boundedness of characteristic yields the reduction of number of exceptional values in $D(A_1, A_2, \varepsilon)$.

*) $U(P, \varepsilon)$ is the ε-neighborhood of P.