70. *Some Applications of the Functional-Representations of Normal Operators in Hilbert Spaces. X*

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In this paper we shall treat of some applications of Theorems 2 and 3 established in the first paper of the same title as above [1].

Definitions and preliminaries. Let M be an arbitrarily prescribed positive constant; let $\{\lambda_\nu^{(\omega)}\}_{\nu=1,2,3,\dots}$ be any infinite sequence of complex numbers with multiplicities properly counted such that $\sup_\nu|\lambda_\nu^{(\omega)}|\leqq M$; let c_ω be any finite complex number, not zero; let $\{\varphi_\nu^{(\omega)}\}_{\nu=1,2,3,\dots}$ and $\{\psi_\mu^{(\omega)}\}_{\mu=1,2,3,\dots}$ both be incomplete orthonormal infinite sets in the complex abstract (complete) Hilbert space $\mathfrak{H}$ which is separable and infinite dimensional; let us suppose that these two orthonormal sets are mutually orthogonal and determine a complete orthonormal system in $\mathfrak{H}$; and let $(\beta_{ij}^{(\omega)})$ be a bounded normal matrix-operator with $\sum_{j=1}^{\infty}|\beta_{ij}^{(\omega)}|^2\neq|\beta_{ii}^{(\omega)}|^2$, $i=1,2,3,\cdots$, in Hilbert coordinate space l_2. Then, as already shown [3], the operator $\widetilde{N}_\omega$ defined by

$$\widetilde{N}_\omega=\sum_{\nu=1}^{\infty}\lambda_\nu^{(\omega)}\varphi_\nu^{(\omega)}\otimes L_{\varphi_\nu^{(\omega)}}+c_\omega\sum_{\mu=1}^{\infty}\Psi_\mu\otimes L_{\psi_\mu^{(\omega)}}\quad\Big(\Psi_\mu=\sum_{j=1}^{\infty}\beta_{\mu j}^{(\omega)}\psi_j^{(\omega)}\Big)$$

is a bounded normal operator with point spectrum $\{\lambda_\nu^{(\omega)}\}$ in $\mathfrak{H}$ such that its continuous spectrum is not empty, its norm is given by $\max(\sup_\nu|\lambda_\nu^{(\omega)}|, |c_\omega|\cdot\|(\beta_{ij}^{(\omega)})\|)$, and $\varphi_\nu^{(\omega)}$ is an eigenelement of $\widetilde{N}_\omega$ corresponding to the eigenvalue $\lambda_\nu^{(\omega)}$; and if such $M, c_\omega, \{\varphi_\nu^{(\omega)}\}, \{\psi_\mu^{(\omega)}\}$, and $(\beta_{ij}^{(\omega)})$ as above are appropriately chosen, conversely any bounded normal operator with point spectrum $\{\lambda_\nu^{(\omega)}\}$ and nonempty continuous spectrum in $\mathfrak{H}$ is expressible by such a series of linear functionals $L_{\varphi_\nu^{(\omega)}}, L_{\psi_\mu^{(\omega)}}$ as above. On the assumption that M is fixed, we now denote by $\mathfrak{N}(M)$ the class of bounded normal operators $\widetilde{N}_\omega$ for all those $\{\lambda_\nu^{(\omega)}\}, c_\omega, \{\varphi_\nu^{(\omega)}\}, \{\psi_\mu^{(\omega)}\}$, and $(\beta_{ij}^{(\omega)})$ which satisfy the above conditions respectively. Moreover, for any $\widetilde{N}\in\mathfrak{N}(M)$ we denote by $\varDelta(\widetilde{N})$ the continuous spectrum of $\widetilde{N}$, by $\varDelta^+(\widetilde{N})$ the set of all those accumulation points of the point spectrum $\{\lambda_\nu\}$ of $\widetilde{N}$ which do not belong to $\{\lambda_\nu\}$ itself, by $\varDelta^-(\widetilde{N})$ the set $\varDelta(\widetilde{N})-\varDelta^+(\widetilde{N})$, and by $\{K(\zeta)\}$ the complex spectral family of $\widetilde{N}$. Then, as already pointed out in one of the preceding papers [2], $\widetilde{N}[I-K(\varDelta^-(\widetilde{N}))]$ is a bounded normal operator whose point spectrum and continuous spectrum are given by $\{\lambda_\nu\}$ and $\varDelta^+(\widetilde{N})$ respectively.