69. *On the Explanation of Observables and States*

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§1. **Introduction.** In the previous paper [1], three kinds of multiplications of operator valued functions are given. But the differences among them are very delicate and important. Here, giving the exact definitions of the testing functions and mollifiers, the differences among these multiplications are discussed.

Since the multiplications used in axiomatic relativistic quantum field theory is (2) in [1], the non local field appears [7]. Here for the purpose of the construction of local observables appearing in Wightman function, the functional integration is used [6].

On the other hand Von Neumann has constructed the direct product space to represent the state vectors. But, between this and true state vectors' space there are following differences [2]:

(1) True space of state vectors is not a Hilbert space but a space consisting of vectors with unit length.

(2) In Von Neumann's direct product space, the treatment of the states with infinite phase amplitude is not necessarily faithful to the treatment of the state vectors.

Hence, in this paper, by considering the formal meaning of vectors contained in Von Neumann's direct product space, the useful new state is constructed by the Gelfand construction in [3]. Furthermore, the true character of this constructed states is shown.

§2. **The relation among the three kinds of multiplications.** Let's also use the most of the notations and definitions found in [1], [3].

In [1], the three kinds of multiplications of the operator valued functions have been defined, and the differences among them also have been investigated. Here, let's give the deeper consideration to the differences among them.

For these definitions in [1], the mollifiers, the testing functions or both of them are always used.

Suppose that $\varphi(x)$ is represented by the triplet $[\varphi(x), \{\rho(x)\}, \{f(x)\}]$ and the multiplications are defined in the set of these triplets. Here $\varphi(x)$ is an operator valued function, $\{\rho(x)\}$ is the set of mollifiers and $\{f(x)\}$ is the set of testing functions.

For these definitions of multiplications $\{\rho(x)\}$ and $\{f(x)\}$ are not necessarily used at the same time.

Namely in the multiplication of (1) in [1] $\varphi(x)$ and $\{f(x)\}$ are