85. Some Applications of the Functional-Representations of Normal Operators in Hilbert Spaces. XI

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In this paper we are again concerned with the problem of applying Theorem 3 [cf. Proc. Japan Acad., Vol. 38, No. 6, 267–268 (1962)] from a different point of view.

Theorem 28. Let M be a positive constant; let $\mathfrak{H}$, (β_{ij}), and $\mathfrak{F}(M)$ be the same notations as those defined in the preceding paper; let $\{\lambda_\nu\}_{\nu=1,2,3,\cdots}$ be an arbitrarily prescribed infinite sequence of complex numbers (counted according to the respective multiplicities) such that $\sup_\nu|\lambda_\nu|\leqq M$; let $\{\varphi_\nu\}_{\nu=1,2,3,\cdots}$ and $\{\psi_\mu\}_{\mu=1,2,3,\cdots}$ both be incomplete orthonormal systems which are mutually orthogonal and determine a complete orthonormal system in $\mathfrak{H}$; let c be an arbitrarily given complex number, not zero; let N be the bounded normal operator defined by

$$N=\sum_{\nu=1}^{\infty}\lambda_\nu\varphi_\nu\otimes L_{\varphi_\nu}+c\sum_{\mu=1}^{\infty}\Psi_\mu\otimes L_{\psi_\mu}\quad (\Psi_\mu=\sum_{j=1}^{\infty}\beta_{\mu j}\psi_j);$$

let Γ be a rectifiable closed Jordan curve, positively oriented, such that the disk $|\lambda|\leqq\max[M,|c|\cdot||(\beta_{ij})||]$ lies in the interior of Γ itself; and let $(\beta_{ij})^n=(\beta_{ij}^{(n)})$, $(n=0,1,2,\cdots;\beta_{ij}^{(0)}=0$ for $i\neq j$; $\beta_{jj}^{(0)}=1$ for $j=1,2,3,\cdots;\beta_{ij}^{(1)}=\beta_{ij})$, for convenience. Then, for the ordinary part $R_\omega(\lambda)=\sum_{n\geqq 0}a_\omega^{(n)}\lambda^n$, $(|\lambda|<\infty)$, of any $S_\omega(\lambda)\in\mathfrak{F}(M)$,

$$(26)\quad \frac{1}{2\pi i}\int_\Gamma S_\omega(\lambda)(\lambda I-N)^{-1}d\lambda=\sum_{\nu=1}^{\infty}R_\omega(\lambda_\nu)\varphi_\nu\otimes L_{\varphi_\nu}+\sum_{\mu=1}^{\infty}\sum_{n\geqq 0}a_\omega^{(n)}c^n\Psi_\mu^{[n]}\otimes L_{\psi_\mu}\equiv T\quad (i=\sqrt{-1}),$$

where $\Psi_\mu^{[n]}=\sum_{j=1}^{\infty}\beta_{\mu j}^{(n)}\psi_j$ and the linear functional-series T on the right-hand side is a bounded normal operator with point spectrum $\{R_\omega(\lambda_\nu)\}_{\nu=1,2,3,\cdots}$ in $\mathfrak{H}$. Moreover the eigenspace of T corresponding to the eigenvalue $R_\omega(\lambda_\nu)$ coincides with that of N corresponding to the eigenvalue λ_ν.

Proof. Since, by hypotheses, (β_{ij}) is a bounded normal matrix-operator with $\sum_{j=1}^{\infty}|\beta_{ij}|^2\neq|\beta_{ii}|^2>0$, $(i=1,2,3,\cdots)$, in Hilbert coordinate space l_2, the point spectrum of N is surely given by $\{\lambda_\nu\}$, as already demonstrated before [cf. Proc. Japan Acad., Vol. 39, No. 10, 743–748 (1963)]. Moreover, by hypotheses, all the singularities of $S_\omega(\lambda)$ and the (point and continuous) spectra of N are wholly contained in the interior of Γ. By reference to Theorem 3, we have therefore