81. *On a Definition of Singular Integral Operators. I*

By Hitoshi KUMANO-GO
Department of Mathematics, Osaka University
(Comm. by Kinjirô KUNUGI, M.J.A., June 12, 1964)

Introduction. The theory of singular integral operators of A. P. Calderón and A. Zygmund [1] has been applied to the various problems in partial differential equations, since A. P. Calderón [2] succeeded in proving the general theorem for the uniqueness of solutions of the Cauchy problem by using this theory. S. Mizohata in the notes [7], [8], and [9] proved the many interesting theorems for the uniqueness by modifying the notion of singular integral operators, M. Yamaguti [12] applied these operators to the existence theorem of solutions of the Cauchy problem for hyperbolic differential equations and M. Matsumura [6] applied to the existence and non-existence theorems of local solutions of the general equations.

In the note [4] we introduced singular integral operators of class $C^m_{\mathfrak{m}}$ and proved the theorems of [7] and [8] by a unified method, and also in [5] we generalized the theorem of [9] by applying the operators of this class.

In the present note we shall give a definition of singular integral operators which governs operators of class $C^m_{\mathfrak{m}}$, and prove that the main theorems relating to operators of [1] hold for the present operators. In this theory we do not require the homogeneity of the symbol $\sigma(H)(x, \eta)$ in η (Definition 4), but assume the analyticity in η. The technique of almost all the proofs is based on [10] and [12], and the exposition is self-contained. I thank here my colleague K. Ise for helpful discussions.

1. Definitions and lemmas. Let $x=(x_1,\cdots,x_n)$ be a point of Euclidean n-space R^n_x, $\xi=(\xi_1,\cdots,\xi_n)$ be a point of its dual space E^n_ξ and $\alpha=(\alpha_1,\cdots,\alpha_n)$ denote a real vector whose elements are non-negative integers.

We shall use the notations:

$\alpha!=\alpha_1!\cdots\alpha_n!$, $|\alpha|=\alpha_1+\cdots+\alpha_n$, $x\cdot\xi=x_1\xi_1+\cdots+x_n\xi_n$,

$D_x=(D_{x_1},\cdots,D_{x_n})=(\partial/\partial x_1,\cdots,\partial/\partial x_n)$, $x^\alpha=x_1^{\alpha_1}\cdots x_n^{\alpha_n}$, $D_\xi=(\cdots$, etc.

The terminology employed is that of L. Schwarz [11].

The Fourier transform $\mathfrak{F}[u](\xi)=\hat{u}(\xi)$ of a function $u\in L^2_x$ is defined by

$$\mathfrak{F}[u](\xi)=\frac{1}{\sqrt{2\pi}^n}\int e^{-\sqrt{-1}x\cdot\xi}u(x)\,dx.$$