110. *On the Definition of Functional Integrals*

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§1. **Introduction.** Functional integral is one of the powerful tool in quantum field theory or stocastic process [1–2]. The trial to define it exactly and naturally has been done by K. O. Friedricks [1]. But his definition is still restricted by the usual Hilbert space $\mathfrak{H}$. Many definitions of it are given, but each of them is not sufficient and we must show the more suitable new definition, because they cannot describe the important integral skilfully. Therefore, the integral of functions defined in the Hilbert space (or in its extension) is not necessarily used in usual, though it is one of the powerful tools.

Here, let's investigate precisely the definition in [1–2] and give the generalized definition which is faithful to the following Example 1. Here, Example 1 is the most basic one showing the natural explanation of this integral. Our method is one corresponding to the continuous representation of states [5–6].

On the other hand, Feynman integral (a sort of singular functional integral) is one of the important purpose of this research. We have already succeed to define it by using E. R. Integral which is the most general singular integral. For functional, this mild integral is effective specially [7], [1].

In the next paper, we will show it. In this paper, we give the relation between Feynman integral and our generalized definition for preliminalies.

§2. **Definition of the functional integral.** Let $\mathfrak{H}$ denote the space of the real square integrable functions defined on the real axis. (We may change to complex valued functions easily.)

Let $f(\xi(s))$ (for $\xi(s)$, $-\infty<s<+\infty$) denote the real valued functional defined on $\mathfrak{H}$ and $I[f(\xi(s))]$ denote the integral of $f(\xi(s))$, namely $I[f(\xi(s))]=\int_{\mathfrak{H}} f(\xi(s))\, dm(\xi(s))$.

Here we show the most elementary example of the functional integral, and give the generalized definition which is faithful to this example.

Example 1. Let $\mathfrak{H}$ denote the space of sequences $\{x_1, x_2, \cdots\}$ with the property $\Sigma_{i=1}^{\infty}|x_i|^2<+\infty$. (For example, the sequence of Fourier coefficients for some fixed base.)

$$I[f(x_1, x_2, \cdots)]=\lim_{n\to\infty}\int\cdots\int(1/\sqrt{2\pi}\,\sigma)^n \exp\{-\Sigma_{i=1}^{n} x_i^2/2\sigma^2\}\times$$