109. *Extension of a Certain C* Algebra*

By Hideo YAMAGATA

(Comm. by Kinjirô KUNUGI, M.J.A., Sept. 12, 1964)

§1. **Introduction.** All field operators such as creation operator, annihilation operator and the values of field function obtained by the cut off process are unbounded operators. Then C^* algebra consisted of all bounded operators is not necessarily sufficient as the set of observables. For the investigation of the various characters of field operators, and for using the results of many researches with respect to C^* algebra effectively, its suitable extension is needed. The weak topology used by R. Haag and D. Kastler in [1] becomes to one powerful tool to extend C^* algebra by its suitable use. In this paper, we don't device the suitable use of the weak topology, but we will consider the meaning of the domain of selfadjoint unbounded operators, and show the limitation of the most usual weak extension of this C^* algebra from deeper view point than [2].

On the other hand, another extension using E. R. Integral has been shown in [2]. The generalized mathematical expectation by using A-integral (equivalent to E. R. Integral) has been already defined by Kolmogorov in the most primitive form [7]. But quantized solution of the Klein Golden equation, etc. cannot be treated by this method. Here, we will show the more extended concept of the observables by using n dimensional E. R. Integral and give the definition of the observables containing the above solutions, etc.

§2. **Unbounded operators produced by Weak extension.** Here, we only consider the linear operator T densely defined in $\mathfrak{H}$. The definition of unbounded operator is the following usual one.

Definition 1. If there is not a fixed bounded number C such that $\|T\varphi\|\leqq C\|\varphi\|$ for all φ contained in the domain, we call this operator T an unbounded linear operator.

From the definition of the usual weak topology, we classify the set of unbounded linear operators to the following two types:

(1) the domain of T is $\mathfrak{H}$,

(2) the domain of T is purely contained in $\mathfrak{H}$.

Let's show the examples belonging to each class.

Example 1. Let $\{e_n\}$ be a base in $\mathfrak{H}$, let D be the set of the finite linear aggregate of e_n and let T be a linear operator defined in the set D with the property $Te_n=ne_n$.

At the first step, using this D, let's classify $\mathfrak{H}$ and construct the space of the classes $\mathfrak{H}/D$.