# 105. A Remark on a Construction of Finite Factors. II

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1. In the previous paper [2], we proved that the crossed product $G\otimes\mathcal{A}$ of a finite von Neumann algebra $\mathcal{A}$ by a group $G$ of outer automorphisms has the property $Q$ (Definition 2, in the below), only if $G$ is amenable, and that the factor constructed by an enumerable ergodic $m$-group $G$ on a measure space by the method due to Murray and von Neumann [4] is a continuous hyperfinite factor only if $G$ is amenable.

In the present note, we shall show that the crossed product of a finite von Neumann algebra with the property $Q$ by an amenable group $G$ of outer automorphisms has the property $P$ in the sense of J. T. Schwartz [5] (Definition 1, in the below), and that the factor constructed by an enumerable ergodic amenable $m$-group $G$ and a measure space by the method due to Murray and von Neumann has the property $P$.

We shall use the terminology due to Dixmier [3] and the previous note [2] without further explanations.

2. In the first place, we state some properties of the operator Banach mean defined in [1]. Let $G$ be a discrete group and $L^\infty(G)$ the algebra of all bounded complex-valued functions on $G$. We shall denote a Banach mean on $L^\infty(G)$ by $\int_G x(g)dg$. A group with a Banach mean will be called *amenable*. Let $\{T_g;\, g\in G\}$ be an operator family which is uniformly bounded on a Hilbert space. If $G$ is amenable, then

$$[x\,|\,y]=\int_G (T_g x\,|\,y)dg$$

is a bounded bilinear form on the Hilbert space. Hence, there exists a unique bounded operator $T$ such that $[x\,|\,y]=(Tx\,|\,y)$. Then we shall call $T$ the *operator Banach mean* on $G$ and write it by

$$T=\int_G T_g\, dg.$$

It is proved in [1] that the operator Banach mean satisfies the following properties:

a) $\int[\alpha T_g+\beta S_g]\,dg=\alpha\int T_g\,dg+\beta\int S_g dg,$