# 99. *The Area of Discontinuous Surfaces*

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**1. Introduction.** Let us use the term *rectangle* synonymously with nondegenerate closed interval of the Euclidean plane $\boldsymbol{R}^2$. By a *nonparametric summable surface* on a rectangle we understand a surface of the form $z=F(x, y)$, where $F$ is a summable function defined on the rectangle and assuming finite real values. For brevity, such a surface will often be referred to as an NS surface.

A few authors have already treated the area theory of NS (or more general) surfaces, Cesari [1] and Goffman [3] being representative. The greater part of this paper is concerned with a further contribution to the theory, in which another definition of area will be given to NS surfaces and will be shown equivalent to those of Cesari and Goffman.

We shall apply then our leading idea to *parametric summable surfaces* (§ 6), to obtain a concept of area which, in the special case of parametric continuous surfaces, coincides with the Lebesgue area.

If one seeks to generalize the various results of the existing area theory so as to be valid for parametric summable surfaces, there arise in a natural way a number of research problems. Some of them will be stated toward the end of the paper.

**2. Area of nonparametric summable surfaces.** For any continuous function $G$ on a rectangle $I_0$, the Lebesgue area of the surface $z=G(x, y)$ will be denoted by $S(G)$ or $S(G; I_0)$, as in [Saks 4]. If $G^*$ is another continuous function on $I_0$ and $E$ is any nonvoid subset of $I_0$, the symbol $\rho(G, G^*; E)$ will mean the ordinary distance on $E$ between the two functions, i.e. the supremum of $|G(w)-G^*(w)|$ for $w \in E$. If $E$ is the void set, the same symbol is understood to vanish.

Let $I=[a_1, b_1; a_2, b_2]$ be a rectangle and let $h$ stand for the positive numbers $<2^{-1}\min(b_1-a_1, b_2-a_2)$. We shall write, in the sequel,

$$I_h=[a_1+h, b_1-h; a_2+h, b_2-h].$$

Given on $I$ a finite summable function $F$, we understand by the *integral mean* of $F$ (for squares of side-length $2h$), the function

$$F_h(x, y)=\frac{1}{4h^2}\int_{-h}^{h}\int_{-h}^{h}F(x+u, y+v)dudv,$$

where the point $\langle x, y\rangle$ ranges over the rectangle $I_h$. It is well known that $F_h$ is then a continuous function on $I_h$.