140. Some Applications of the Functional-Representations of Normal Operators in Hilbert Spaces. XIV

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Let $T(\lambda)$ be such a function as was defined in the preceding paper (that is, in Part XIII) [cf. Proc. Japan Acad., Vol. 40, No. 7 (1964)]. In the present paper we shall derive another formula of the expansion of $T(\lambda)$ and shall discuss some of its applications.

Since, as we have already shown in Part XIII, the form of the expansion by series or by integrals of $T(\lambda)$ outside a suitably large circle with center at the origin is exactly similar to that of the function $S(\lambda)$ in Theorem 1 [cf. Proc. Japan Acad., Vol. 38, No. 6, 265–267 (1962)], we can establish the following propositions for the question as to whether the ordinary part of $T(\lambda)$ is a polynomial in λ or a transcendental integral function.

Proposition A. Let $T(\lambda)$, $\{\lambda_\nu\}_{\nu=1,2,3,\dots}$, D_j, $(j=1,2,3,\cdots,n)$, and ρ be the same notations as those used in Theorem 33 respectively, and $M_T(\rho,0)$ the maximum modulus of $T(\lambda)$ for all the points on the circle $|\lambda|=\rho$. Then a necessary and sufficient condition that the ordinary part of $T(\lambda)$ be a polynomial in λ of the degree less than or equal to d is that there exist a positive constant K and a suitably large number σ such that

$$M_T(r,0)\leqq Kr^d$$

for every r with $\max[\sup_\nu|\lambda_\nu|, \max_j(\max_{z\in D_j}|z|)]<\sigma<r<\infty$ [cf. Proc. Japan Acad., Vol. 38, No. 10, 706–707 (1962)].

Proposition B. Let $T(\lambda)$, ρ, and $M_T(\rho,0)$ be the same notations as above respectively. Then a necessary and sufficient condition that the ordinary part of $T(\lambda)$ be a transcendental integral function of the order $d>0$ is that

$$\varlimsup_{\rho\to\infty}\frac{\log\log M_T(\rho,0)}{\log\rho}=d>0 \quad \text{[cf. loc. cit., 708–709].}$$

In fact, these propositions can be shown by replacing $S(\lambda)$ in the proofs of Theorems 13 and 14 by $T(\lambda)$. As for Proposition A, however, we can simplify it by making use of the following theorem derived from the already established expansion by series of $T(\lambda)$ outside a suitably large circle with center at the origin.

Theorem 38. Let $T(\lambda)$ and ρ be the same notations as those used in Theorem 33 respectively, and let