138. *E. R. Functional Integrals*

By Hideo YAMAGATA

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§1. **Introduction.** In order to define Feynman integral exactly, the extension of the following spaces is required in functional integral: (1) the functional's domain $\mathfrak{H}$, (2) the set of integrable (not necessarily bounded) functionals defined on $\mathfrak{H}$. To perform the extension (1), outer Hilbert space [1] or neuclear space [2] is already constructed, because we cannot define even the completely additive Gauss measure in $\mathfrak{H}$. But the concrete meaning of these extended spaces are not yet obvious, and it is difficult to clarify this. Hence let's show here another more concrete and more delicate extension of $\mathfrak{H}$ by using E. R. integral without showing the relation between our extension and the neuclear space etc. The meaning of this extended space in Feynman integral is the increase of the considerable path in quantum field theory. It seems to us that this extended space gives the negative effect to the extension (2). For our purpose it needs to compensate this negative effect by the suitable use of both extensions (1) and (2). Furthermore, the extension (2) in Feynman integral permits us to use the more singular potential.

Here, using E. R. integral as the most general singular integral, the above extensions (1) and (2) for general functional integral is performed in the most wide meaning. Furthermore, the type of singularities in Feynman integral is investigated, and the possibility of the definition of E. R. Feynman integral constructed by the extensions (1) and (2) is shown. Recently, the equivalence between the primitive E. R. integral and A-integral by A. И. Kolmogoroff has been proved. A-integral has very simple form [8], but in order to construct the more wide organized extension of usual integral, E. R. form is more useful than A-integral.

§2. **Definition of E. R. functional integral.** Afterwards, we will use the same notations as one used in [5]. In this paragraph, we will extend the set of integrable functionals $\overline{C}$ defined in [5]. For its preliminaries, let's show an explanation about the concept of cylinder functional appearing in the usual definition of functional integral. Namely cylinder functional can be considered as the step functional which is constant in the set of $\xi(s)$ such that $P_j^{(N)}(\xi(s))=P_j^{(N)}(\xi^0(s))$ for any j, fixed D_N and fixed $\xi^0(s)$. The original $\overline{C}$ in [5] is the set of the functional f constructed by the convergent sequences of step functionals in $L^1(\mathfrak{H})$. Required improvements are the following two.