136. On Quasi-Montel Spaces

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1. In the theory of topological linear spaces, many properties of Montel spaces have been studied. In this paper, we shall investigate the properties of the spaces having weaker condition than Montel spaces, called "quasi-Montel spaces". Throughout this paper, terminology and notation are the same as in [1], if nothing otherwise is mentioned. For example, a Montel space means a locally convex separative topological linear space in which every bounded subset is relatively compact, and which is not necessarily tonnelé.

Definition. *We say that a locally convex separative topological linear space E is a quasi-Montel space, if and only if each convex weakly compact ($\sigma(E, E')$-compact) subset is compact for the original topology of E.*

Obviously, each Montel space is a quasi-Montel space.

Theorem 1. *In order that a locally convex separative topological linear space E is a Montel space, it is necessary and sufficient that E is a semi-reflexive[1] quasi-Montel space.*

Proof. Necessity is trivial.

Sufficiency: For any closed bounded subset A of E, there is a convex closed bounded subset B containing A. From the semi-reflexivity, B is a weakly compact subset. So B is compact, because E is a quasi-Montel space. Therefore A is also compact.

Theorem 2.

(a) *A subspace of a quasi-Montel space is a quasi-Montel space.*

(b) *A product space of quasi-Montel spaces is a quasi-Montel space.*

(c) *A direct sum of quasi-Montel spaces is a quasi-Montel space.*

(d) *A strict inductive limit of countable many quasi-Montel spaces is a quasi-Montel space.*

Proof. (a) Let E be a quasi-Montel space and F be a subspace of E. Each convex weakly compact ($\sigma(F, F')$-compact) subset A of F is convex weakly compact ($\sigma(E, E')$-compact) in E. As E is a quasi-Montel space, A is a compact subset of E. Therefore A is also compact in F.

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1) We say that a topological linear space E is semi-reflexive, if each continuous linear functional on E' is continuous for $\sigma(E', E)$-topology.