134. *The Number of Irreducible Components of an Ideal and the Semi-Regularity of a Local Ring*

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(Comm. by Zyoiti SUETUNA, M.J.A., Oct. 12, 1964)

Let Q be a local ring with the maximal ideal $\mathfrak{m}$, and $\mathfrak{q}$ be an $\mathfrak{m}$-primary ideal of Q. Then it is known that the number n of irreducible components of $\mathfrak{q}$ is equal to the length $L_Q(\mathfrak{q}:\mathfrak{m}/\mathfrak{q})$ of Q-module $\mathfrak{q}:\mathfrak{m}/\mathfrak{q}$, which was defined as the index of reducibility of $\mathfrak{q}$ by D. G. Northcott in his paper [1]. In the same paper, he proved that, *if Q is semi-regular, then the index of reducibility of an $\mathfrak{m}$-primary ideal generated by a system of parameters depends only on Q, and not on the choice of the system of parameters* (Theorem 3, [1]). Gröbner's theorem, i.e. *in a regular local ring, any ideal which can be generated by a system of parameters is irreducible*, is a special case of this theorem.

On the other hand, it is known that, in a local ring Q, *if the index of reducibility of an $\mathfrak{m}$-primary ideal generated by a system of parameters is equal to* 1 *constantly* (i.e. *if every ideal generated by a system of parameters is irreducible*), *then the ring Q is semi-regular* (see [2]). However it should be noticed that the condition of this proposition is not a sufficient condition for the regularity of Q.

Concerning these results, the following question may be raised:

Can we conclude that the local ring Q is semi-regular if the index of reducibility of an $\mathfrak{m}$-primary ideal generated by a system of parameters is equal to some constant which is not necessarily 1?

The main purpose of this paper is to answer this question.

Now we shall begin by proving the following theorem:

Theorem 1. *Let Q be a local ring of dimension d, and $\mathfrak{m}$ be its maximal ideal. If $\mathfrak{m}$ is generated by d or $d+1$ elements, and if the index of reducibility of an $\mathfrak{m}$-primary ideal generated by a system of parameters is equal to some constant which does not depend on the choice of such an ideal, then every $\mathfrak{m}$-primary ideal generated by a system of parameters is irreducible.*

Proof. If $\mathfrak{m}$ is generated by d elements, then Q is regular, and the conclusion follows (see Gröbner [3] or Northcott [1]).

Now we shall assume that the minimal basis of $\mathfrak{m}$ consists of $d+1$ elements. Then it is easy to see that we can assume that $\mathfrak{m}$ is generated by $u_1, u_2, \cdots, u_d, u_{d+1}$ where $\{u_1, u_2, \cdots, u_d\}$ is a system of

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