158. On the Spectra of Uniformly Increasing Mappings

By Sadayuki YAMAMURO*)

(Comm. by Kinjirô KUNUGI, M.J.A., Nov. 12, 1964)

Let E be a real Banach space, G be an open set and $\bar{G}$ be its closure.

In [2], we have given the following definition:

A mapping f *of* $\bar{G}$ *in* E *is said to be* $(\varepsilon_0, \delta_0)$*-uniformly increasing at* $a \in G$ *if*

(i) $a+x \in \bar{G}$ *if* $||x|| \leqq \delta_0$;

(ii) $||f_a(x)-\alpha x|| \geqq \varepsilon_0 ||x||$ *for any non-positive number* α *and any element* x *such that* $||x|| \leqq \delta_0$, *where* $f_a(x)=f(a+x)-f(a)$.

The purpose of this paper is to prove the following

Theorem. *Assume that*

1. $F(x)$ *is a completely continuous mapping of* $\bar{G}$ *in* E;
2. $F(a)=\lambda_0 a$ *for some* $\lambda_0 \neq 0$ *and some* $a \in G$;
3. $f(x)=x-\frac{1}{\lambda_0}F(x)$ *is* $(\varepsilon_0, \delta_0)$*-uniformly increasing at* a.

Then, we have that

1°. a *is an isolated fixed point of* $\frac{1}{\lambda_0}F(x)$;

2°. *For any* λ *such that* $|\lambda-\lambda_0| < \min.\left\{|\lambda_0|, \frac{|\lambda_0|\varepsilon_0\delta_0}{||a||+\delta_0}\right\}$, *there exists* x_λ *such that*

$$F(x_\lambda)=\lambda x_\lambda \quad \text{and} \quad ||x_\lambda-a|| \leqq \frac{1}{|\lambda_0|\varepsilon_0}(||a||+\delta_0)|\lambda-\lambda_0|.$$

Remark. A mapping $F(x)$ is said to be *completely continuous on* $\bar{G}$ if it is continuous and the image $F(\bar{G})$ is contained in a compact set.

Proof. 1°. Assume that a is not an isolated fixed point of $\frac{1}{\lambda_0}F(x)$, then there exists a sequence $\{x_n\}$ such that

$$\lim_{n\to\infty} x_n=0 \quad \text{and} \quad F(a+x_n)=\lambda_0(a+x_n).$$

Since $f(x)=x-\frac{1}{\lambda_0}F(x)$, we have

$$f(a+x_n)=(a+x_n)-\frac{1}{\lambda_0}F(a+x_n)=0.$$

Now, since $f(x)$ is $(\varepsilon_0, \delta_0)$-uniformly increasing at a, we have

*) Department of Mathematics, Institute of Advanced Studies, Australian National University.