156. The Role of Mollifiers in S Matrix Theory

By Hideo YAMAGATA

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§ 1. **Introduction.** In order to describe S matrix in the form $S=T\left(\exp i\int g(x)L(x)dx\right)=\sum_{n=0}^{\infty}(i^n/n!)\int T(L(x_1)\cdot L(x_2)\cdots L(x_n))g(x_1)g(x_2)\cdots g(x_n)dx_1\cdots dx_n$, a function $g(x)$ is used. By using the discussions in [4–6], it can be shown that this function $g(x)$ does not necessarily play the role of testing functions but mollifiers. Namely the direct product of the same $g(x)$ contained in (D) cannot construct the dense set in $(D)\otimes(D)\otimes\cdots\otimes(D)$, where (D) is the space consisting of C^∞ functions with compact carrier defined by L. Schwartz [2]. Even in infinite direct product space constructed by (D) the same problem happens. From this it is obvious that the above description of S matrix is very imcomplete. In § 2 we will show this. This result necessarily shows the incompleteness of the description of causality condition, too. Namely our causality condition is effective to only the S matrix described by the form $S(g)$. Furthermore, it is the limit of formulas showing a sort of causality condition which is effective to non local Lagrangian. To describe the causality condition directly, we must use the element of ranked space instead of $g(x)$ [7–8]. We will show these facts in § 3.

§ 2. **The product of distributions in S matrix theory.** Afterward, we use the following notations.

Let $T(u(x)u(y))$ denote the product

$$T(u(x)u(y))=\begin{cases} u(x)u(y) & \text{for } x^0>y^0 \\ \pm u(y)u(x) & \text{for } x^0<y^0. \end{cases}$$

(For Bose operators, the sign $+$ is used, and for Fermi operators the sign $-$ is used.) This product is called chronological product or T-product.

Let $T(u(x)\otimes u(y))$ denote the direct product

$$T(u(x)\otimes u(y))=\begin{cases} u(x)\otimes u(y) & \text{for } x^0>y^0 \\ \pm u(y)\otimes u(x) & \text{for } x^0<y^0. \end{cases}$$

(For Bose operators the sign $+$ is used, and for Fermi operators the sign $-$ is used.) This direct product is called chronological direct product or T direct product.

Let (D) denote the space of C^∞ functions with compact carrier which has the topology defined by L. Schwartz in [2], $\prod_{i=1}^{\infty}\otimes(D)$ denote the infinite direct product of (D), and D_∞ denote the closure of the linear aggregate of the elements in $\prod_{i=1}^{\infty}\otimes(D)$ by means of