154. Locally Convex Metrizable Topologies which Make a Given Vector Subspace Dense

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1. The current theory of quantum field encounters with a difficulty arized from the fact that the orthogonality of the specific vector subspaces of the space of physical states denies the expansion of certain physical states.[1] It might be possible, however, to avoid this difficulty by changing the topology of the space of physical states. In the problem of this possibility, it is fundamental to show the existence of a topology having the property that a given vector subspace is dense in the whole space; moreover it will be desirable that the topology is locally convex and metrizable.[2]

The purpose of this paper is to give a condition under which the existence of such a topology is ensured.

The author is indebted to Professor T. Ishihara for suggesting this problem.

2. We shall concern exclusively with complex vector spaces. But every result obtained in what follows remains valid for real vector spaces.

Let E be a vector space, and let M be a vector subspace of E. A vector subspace N of E is called an *algebraic supplement* of M in E, if $M+N=E$ and $M \frown N=\{0\}$. A subset A of E is said to be *linearly independent* if a (finite) linear combination $\sum_{i=1}^{n} \lambda_i x_i$, where $x_i \in A$ for each i and $x_i \neq x_j$ for $i \neq j$, is 0 only when each λ_i is zero. By a *base* of E, we mean always a Hamel base of E, that is, a maximal linearly independent subset of E. We denote by $\dim (E)$ the dimension of E, i.e. the cardinal number of a base of E, and by $\mathrm{codim}\,(M)$ the codimension of M in E.

A pair (E, E') of vector spaces E and E' is called a *dual system* if a bilinear functional $\langle \cdot, \cdot \rangle$ on the product space $E \times E'$ is assigned. A dual system (E, E') is said to be *separated* if it satisfies the following two conditions:

1° If $\langle x, x' \rangle = 0$ for all $x' \in E'$, then $x=0$.
2° If $\langle x, x' \rangle = 0$ for all $x \in E$, then $x'=0$.

A dual system (E, E') which satisfies the condition 1° (resp. 2°) is

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1) A brief survey of this circumstance can be found in T. Ishihara [1].
2) Cf. T. Ishihara [1].