151. *Stability in Topological Dynamics*

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1. **Introduction.** In [1], I. Bendixson established many fundamental theorems concerning the flow in the plane defined by a system of differential equations, one of which gives a criterion for the both sense stability of an isolated singular point p: the point p is stable in both senses, positive and negative, if and only if p is a center (in the sense of Bendixson). It has been proved that this criterion is valid if we replace the point p by a simply connected compact invariant set M, isolated from singular points [2]. Since a one-dimensional compact invariant set disjoint from singular points is always a closed orbit, the criterion is transformed as follows: a simply connected compact invariant set M isolated from singular points is stable in both senses, if and only if, in every neighborhood U of M, there exists a non-dense closed invariant set which separates M and the complement of U.

In an earlier paper [3], Ura proved that this transformed condition is also necessary and sufficient for the both sense stability of a compact invariant set of an abstract continuous flow whose phase space is an arbitrary topological space, under only one condition that the space is locally compact, and showed also that the assumptions of M to be isolated from singular points and to be simply connected can also be omitted.

The first object of the present note is to extend again the criterion to more general systems, i.e., to continuous transformation groups whose phase spaces and groups are arbitrary. We shall show that this is done under very slight restrictions on the phase spaces and the phase groups.

As we shall see, the notions related to stability-additivity defined by Ura [3], are also introduced in a very natural way to general transformation groups as to continuous flows; and the second object of this note is to see that general transformation groups have stability-additivity under some slight conditions on the phase spaces and the phase groups.

Remark: All terminologies concerning topology are referred to Bourbaki [4].

2. **Basic notations.** A continuous transformation group is a triple (X, T, π), where X is a topological space, T is a topological