150. A Note on Riemann's Period Relation

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1. Let W be a Riemann surface of infinite genus and $\mathfrak{J}$ the ideal boundary of W. First we consider the following classes of dividing cycles on W.

DEFINITION 1. *A dividing cycle C on W belongs to the class $\mathfrak{D}_h$ of dividing cycles of order at most h when, for $h>1$, C can be written as $C=\sum_{k=1}^{K}\sigma_k$ with some $K\leqslant h$ where σ_k is a closed curve, and for any $i\leqslant K$ $\sigma_1\cdots\sigma_{i-1}, \sigma_{i+1}\cdots\sigma_K$ are homologously independent mod $\mathfrak{J}$. For $h=1$ $\mathfrak{D}_1$ is the class of connected dividing curves.*

DEFINITION 2. *A dividing cycle on W belongs to the class $\mathfrak{D}'_h$ $(\subset\mathfrak{D}_h)$ of dividing cycles of order h, when it is written as $K=h$ in* Def. 1, *i.e.* $\mathfrak{D}'_h=\mathfrak{D}_h-\mathfrak{D}_{h-1}$.

DEFINITION 3. *An exhaustion of W by regular regions (F_n) belongs to the class $\mathcal{E}_h$ of semi canonical exhaustions of at most order h, when it satisfies the following conditions:*

(A) (i) *It is an exhaustion in Noshiro's sense, (cf.* [6], *p.* 50).*)

(ii) *Denoting canonical partition Q of the set of the contours of F_n (cf.* [3]) *by $Q(\partial F_n)=\sum_{i=1}^{m(n)}\Gamma^i_n$, $(\Gamma^i_n\in\mathfrak{D}_h$, $\Gamma^i_n=\sum_{k=1}^{K(n,i)}\sigma^i_{nk}$, and σ^i_{nk} is a closed contour) there exist at least one Γ^i_n such that $\Gamma^i_n\in\mathfrak{D}'_h$.*

(iii) *$\Gamma^i_n\sim\sum_j\Gamma^{ij}_{n+1}$ $(\Gamma^i_n, \Gamma^{ij}_{n+1}\in\mathfrak{D}_h)$ being inner and outer boundary of a component F^i_n of $F_{n+1}-\bar{F}_n$, there is only one component of $F_{n+2}-\bar{F}_{n+1}$ which is adjoined to F^i_n along each Γ^{ij}_{n+1}.*

2. By using Lemma 5 in [2], slit method in [7], and Noshiro's graph in [6], we can prove easily the following

LEMMA 1. *For $h\geqslant 1$, $\mathcal{E}_h\neq\phi$.*

Let D^i_{nk} be an annulus which satisfies the conditions:

(B) (i) D^i_{nk} includes σ^i_{nk} and $\overline{D^i_{nk}}$ is a closed annulus contained in $F_{n+1}-\overline{F_{n-1}}$.

(ii) $\overline{D^i_{nk}}\cap\overline{D^\alpha_{m\beta}}=\phi$ if $n\neq m$ or $i\neq\alpha$ or $k\neq\beta$.

We put $D^i_n=\sum_{k=1}^{K(n,i)}D^i_{nk}$, $D_n=\sum_{i=1}^{m(n)}D^i_n$, $\partial D_n=\beta_n-\alpha_n$. Let M^i_{nk}, M^i_n, and M_n be the moduli of D^i_{nk}, D^i_n, and D_n respectively. We consider a harmonic function u_n in D_n which vanishes on α_n and is equal to M_n on β_n, and the conjugate v_n of u_n satisfies $\int_{\beta_n}dv_n=2\pi$. Putting $u+iv$

*) [p] means the p-th paper in References which are shown at the end of this paper.