149. *A New Theory of Relativity under the Non-Locally Extended Lorentz Transformation Group*

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The present author has established [16] an ameliorated theory of relativity under the group of *extended* Lorentz transformations:

(1) $$\varepsilon_l\bar{\xi}^l=a^l_m(\xi^p)\varepsilon_m\xi^m+\varepsilon_l a^l_0,\ (a^l_0=\text{const.},\ \varepsilon_l=(-1)^{\frac{1}{2}(1+\delta^4_l)}),$$

(2) $$\varepsilon_l\xi^l=\omega^l_\mu(x^\sigma)\varepsilon_\mu x^\mu+\varepsilon_l\omega^l_0,\ (\omega^l_0=\text{const.},\ \varepsilon_\mu=(-1)^{\frac{1}{2}(1+\delta^4_\mu)}),$$

$l, m, \cdots; \lambda, \mu, \cdots=1, 2, 3, 4$; $x^1=x$, $x^2=y$, $x^3=z$, $x^4=ir=ict$; ((x, y, z): rectangular Cartesian coordinates, t=time); $(a^l_m(\xi^p))$ and $(\omega^l_\mu(x^\sigma))$: orthogonal matrices with determinant $\neq 0$; (x^σ), (ξ^l), and $(\bar{\xi}^l)$: II-*geodesic rectangular coordinates* ♯ [1···16]; δ's: Kronecker deltas which are 3-*dimensional extended equiform Laguerre transformations* *, the Einstein space $(R_{\mu\nu}=0)$ $[dS^2=g_{\mu\nu}(x^\sigma)dx^\mu dx^\nu=(-1)^{1+\delta^4_l}\omega^l\omega^l>0,\ g_{\mu\nu}=\omega^l_\mu\omega^l_\nu,\ \omega^l=\omega^l_\mu(x^\sigma)dx^\mu]$ being the map of the Minkowski space (x^σ) by the inverse transformation of the extended Lorentz transformation (2), so that *connection is not necessary* [28]. Thereby the physical interpretations of the geometrical objects were as follows:

(3) dS=action, $\omega^l_\mu(x^\sigma)$=momentum-potential vector; principle of equivalence=invariancy of physical laws under the group * (physical change); "relativity"=referring to ♯; physical lines =II-geodesic curves (straight lines inclusive);

(4) *Hamilton's principle:* $\delta S=0\rightarrow$ equations of motion:

$$\frac{d^2\xi^l}{dS^2}=\frac{d}{dS}\frac{\omega^l}{dS}=\omega^l_\lambda\left\{\frac{d^2x^\lambda}{dS^2}+\Lambda^\lambda_{\mu\nu}\frac{dx^\mu}{dS}\frac{dx^\nu}{dS}\right\}=\omega^l_\lambda\left\{\frac{d^2x^\lambda}{dS^2}+\left\{{\lambda \atop \mu\nu}\right\}\frac{dx^\mu}{dS}\frac{dx^\nu}{dS}\right\}=0,$$

where

(5) $$\Lambda^\lambda_{\mu\nu}=\Omega^\lambda_l\partial_\nu\omega^l_\mu\equiv-\omega^l_\mu\partial_\nu\Omega^\lambda_l,\ [\Omega^\lambda_l\omega^l_\mu=\delta^\lambda_\mu\Longleftrightarrow\Omega^\lambda_k\omega^h_\lambda=\delta^h_k],$$

the (4) representing II-*geodesics* (in the present author's sense) in 4-dimension, which are in 3-dimension "Kanalflächen" enveloped by oriented II-*geodesic spheres* (in the present author's sense) with the particle (x^1, x^2, x^3) as center and a II-*geodesic radius* $r=\int\frac{\omega^4}{dS}dS$. The theory was resumed ([16], p. 623) in the comparison of the present author's theory with the Einstein's, proving the immortal character (comparable with that of the Newton's law) of the former.

In this note, *the said theory will be extended further* by extending the extended Lorentz transformations to "non-locally" ([17–20]) *extended Lorentz transformations. The general procedure consists in considering*