177. *The Riemann Lebesgue's Theorem and its Application to Cut-off Process*

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§1. **Introduction.** In order to avoid the divergence in perturbation for Miyatake Van-Hove model with fixed point source [4–6] various methods are deviced. The most usual and important method of them is the cut-off operation whose true meaning is to use a source with non zero volume (in some meaning). But in cut-off process the causality condition is not satisfied. In quantum field theory, the cut-off operation is used without any hesitation.

Now, let's rewrite this operation to the mathematical form. The periodic function $f(x)$ with period 2π (by considering the physical effect to enclose in a box) can be developed to the series $f(x)=\sum_{n=-\infty}^{\infty} C_n e^{inx}$. Well known Riemann-Lebesgue's theorem is the following

Theorem. *If* $f(x)$ *is contained in* L^1, *then* $\lim_{|n|\to\infty} C_n=0$.

Our requirement is the very troublesome one. Namely, it is desired that the above theorem (by the order $o(1/n)$) is satisfied for $f(x)$ whose definition's domain is the set of isolated points. The function $f(x)$ used in Miyatake Van-Hove model was $\delta(x)$ defined in the interval $[0, 2\pi]$. Afterward, O. Miyatake has used the function $f(x)=\sum_{i=1}^{n} C_i\delta(x_i)$ defined in the interval $[0, 2\pi]$ and has investigated whether this requirement is satisfied or not for this $f(x)$. But the above requirement is not satisfied for these models. Here, as $f(x)$, we will use the finite linear combination of the characteristic functions of nowhere dense perfect set with positive measure appearing in the process tending to δ function or δ-like function (instead of the sum of δ function defined in the set of isolated points), and we will investigate whether the above requirement is satisfied or not. Because, "nowhere dense" corresponds to "isolated" by some meaning, and "perfect" corresponds to "related." The cut-off related to only Riemann-Lebesgue's theorem is called "natural cut-off." For the requirement to the order tending to zero ($o(1/n)$) it seems that "exact cut-off" must be used. For the exact cut-off (by using A-integral) even a sort of countable infinite linear combination of the above characteristic functions is used. The foundation of our consideration is the A-integral representation of distributions (or E. R. integral) by Виноградова Бонди etc. [1–2]. The carrier of the representing function $f(x)$ in A-integral representation has the following properties: