175. On the Absolute Nörlund Summability of a Fourier Series*)

By H. P. DIKSHIT
University of Allahabad, India

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1.1. *Definitions.* Let $\sum a_n$ be a given infinite series and $\{s_n\}$ the sequence of its partial sums. Let $\{p_n\}$ be a sequence of constants, real or complex and let us write

$$P_n=p_0+p_1+\cdots+p_n;\quad P_{-1}=p_{-1}=0.$$

The sequence to sequence transformation:

$$t_n=\sum_{\nu=0}^{n} p_{n-\nu}s_\nu/P_n;\quad P_n\neq 0, \tag{1.1.1}$$

defines the sequence $\{t_n\}$ of Nörlund means[1] of the sequence $\{s_n\}$, generated by the sequence of coefficients $\{p_n\}$. The series $\sum a_n$ is said to be summable (N, p_n) to the sum s if $\lim_{n\to\infty} t_n$ exists and is equal to s, and is said to be absolutely summable (N, p_n), or summable $|N, p_n|$, if $\{t_n\}\in BV$, that is, $\sum_n |t_n-t_{n-1}|\leqq K$.[2]

1.2. Let $f(t)$ be a periodic function with period 2π, and integrable in the sense of Lebesgue over $(-\pi, \pi)$. We assume, without any loss of generality, that the constant term in the Fourier series of $f(t)$ is zero, so that

$$\int_{-\pi}^{\pi} f(t)dt=0 \tag{1.2.1}$$

and

$$f(t)\sim\sum_n (a_n\cos nt+b_n\sin nt)=\sum_n A_n(t). \tag{1.2.2}$$

We write throughout:

$$\phi(t)=\frac{1}{2}\{f(x+t)+f(x-t)\};$$

$$c_{n,k}=\{\sin (n-k)t\}/(n-k);$$

$$R_n=(n+1)p_n/P_n;$$

$$T_n=1/R_n=P_n(n+1)^{-1}/p_n;$$

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1) Nörlund [3].

2) Mears [1].