174. A Tauberian Theorem for (J, p_n) Summability*)

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§1. We suppose throughout that

$$p_n \geq 0, \qquad \sum_{n=0}^{\infty} p_n = \infty,$$

and that the radius of convergence of the power series

$$p(x) = \sum_{n=0}^{\infty} p_n x^n$$

is *1*. Given any series

(1) $$\sum_{n=0}^{\infty} a_n,$$

with the sequence of partial sums $\{s_n\}$, we shall use the notation:

(2) $$p_s(x) = \sum_{n=0}^{\infty} p_n s_n x^n.$$

If the series (2) is convergent in the open interval (0, 1), and if

$$\lim_{x \to 1-0} \frac{p_s(x)}{p(x)} = s,$$

we say that the series $\sum_{n=0}^{\infty} a_n$ or the sequence $\{s_n\}$ is smmable (J, p_n) to s. As is well known, this method of summability is regular. (See, Borwein [1], Hardy [2], p. 80.)

Now we write

$$P_n = p_0 + p_1 + \cdots + p_n, \quad n = 0, 1, \cdots,$$

and

(3) $$t_n = \frac{1}{P_n} \sum_{\nu=0}^{n} p_\nu s_\nu, \quad n = 0, 1, \cdots,$$

with $p_n > 0$. If $\{t_n\}$ is convergent to s, we say that the series $\sum_{n=0}^{\infty} a_n$ or the sequence $\{s_n\}$ is summable $(\bar{N}, p_n)$ to s. This method of summability is also regular, and is equivalent to the Riesz method $(R, P_{n-1}, 1)$. (See, Hardy [2], pp. 57, 86, Jurkat [4], Kuttner [5,6].)

We shall first state the following

Theorem 1. *$(\bar{N}, p_n)$ implies*[1] (J, p_n).

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*) Dedicated to Professor Kinjirô Kunugi for his 60th Birthday.

1) Given two summability methods A, B, we say that A *implies* B if any series or sequence summable A is summable B to the same sum. We say that A is *equivalent* to B if A implies B and B implies A.