171. On a Theorem of Brauer

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The purpose of this paper is to give a simple proof of a theorem of Brauer concerning the principal blocks of characters of finite groups ([4], Theorem 3, see also [3]).

We refer to Brauer [1], [2]; Brauer-Nesbitt [6]; Osima [8], and Curtis-Reiner [7] as for basic concepts and theorems about the blocks of characters of finite groups.

1. Let G be a group of a finite order and let p be a fixed prime number. We choose the algebraic number field Ω such that the absolutely irreducible representations of G can be written with coefficients in Ω. Let $\mathfrak{p}$ be a prime ideal divisor of p in Ω and let $\mathfrak{o}_{\mathfrak{p}}$ be the ring of all $\mathfrak{p}$-integers of Ω, and $\bar{\Omega}$ the residue class field of $\mathfrak{o}_{\mathfrak{p}}$ (mod $\mathfrak{p}$). The residue class map of $\mathfrak{o}_{\mathfrak{p}}$ onto $\bar{\Omega}$ will be denoted by an asterisk; $\alpha \rightarrow \alpha^*$.

If M is a subset of G, we write $|M|$ for the number of elements of M. The centralizer of M in G will be denoted by $C_G(M)$ and the normalizer of M in G by $N_G(M)$.

The group algebra of G over $\bar{\Omega}$ will be denoted by $\Gamma(G)$ and its center by $Z(G)$. If M is a subset of G, we write $[M]$ for the element of $\Gamma(G)$ defined by

$$[M]=\sum_{m \in M} m. \tag{1.1}$$

If $K_1, K_2, \cdots, K_m$ are the conjugate classes of G, the elements $[K_1], [K_2], \cdots, [K_m]$ form a basis of $Z(G)$. Let us denote by $\psi_0, \psi_1, \cdots, \psi_{s-1}$ the distinct linear characters of $Z(G)$. The m (absolutely) irreducible characters $\chi_0=1, \chi_1, \cdots, \chi_{m-1}$ of G are distributed into s blocks $B_0, B_1, \cdots, B_{s-1}$ for p. There exists a one-to-one correspondence between the set of blocks of G and the set of linear characters of $Z(G)$. The block $B_0=B_0(G)$ of G containing the principal character $\chi_0=1$ is called the principal block of G.

Since each primitive idempotent of $Z(G)$ is associated with a block of G, we shall denote by δ_τ the primitive idempotent associated with B_τ. We then have

$$\psi_\tau(\delta_\sigma)=\begin{cases}1, & \tau=\sigma \\ 0, & \tau \neq \sigma.\end{cases} \tag{1.2}$$

If we set