169. *On the Permutability of Congruences on Algebraic Systems*[*)]

By Tsuyoshi FUJIWARA
University of Osaka Prefecture

(Comm. by Zyoiti SUETUNA, M.J.A., Dec. 12, 1964)

K. Shoda discussed in his papers [8], [10], and his book [9] the structure of an algebraic system $\mathfrak{A}$ under the following conditions:

I. $\mathfrak{A}$ has a zero-element, i. e. $\mathfrak{A}$ has a one-element subsystem.

II. Any subsystem of $\mathfrak{A}$ generated by two normal subsystems of $\mathfrak{A}$ is a normal subsystem of $\mathfrak{A}$.

III. Any natural meromorphism between any two residue class systems of $\mathfrak{A}$ is classable.

G. Birkhoff discussed in his book [1] the structure of an algebraic system $\mathfrak{A}$ under the following conditions:

I. $\mathfrak{A}$ has a one-element subsystem.

III*. Any two congruences on $\mathfrak{A}$ are permutable.

K. Shoda told the author that the conditions III and III* are equivalent as stated in the introduction of the author's paper [2]. The conditions III and III* played the important role in their structure theories of algebraic systems.

A. I. Mal′cev proved in his paper [7] the following

Theorem. *Let A be a set of composition-identities with respect to a system V of compositions. Then the following two conditions are equivalent*:

(a) *Any two congruences on any A-algebraic system are permutable.*

(b) *There exists a derived composition $f(\xi, \eta, \zeta)$ of V such that*

$$f(\xi, \eta, \eta) \overset{A}{=} \xi^{1)} \text{ and } f(\xi, \xi, \eta) \overset{A}{=} \eta.$$

Moreover J. Lambek remarked in his paper [6] that *each of the conditions* (a) *and* (b) *is equivalent to the following condition*:

(c) *Any meromorphism between any two A-algebraic systems is classable.*

A. W. Goldie and the author have discussed in the papers [2], [3], [4], and [5] the structure of algebraic systems. The weak permutability and the local permutability of congruences have played the leading role in the theories of A. W. Goldie and of the author.

*) This paper was written while the author held a postdoctorate fellowship of the National Research Council of Canada.

1) $f(\xi, \eta, \eta) \overset{A}{=} \xi$ denotes the fact that $f(x, y, y) = x$ holds for any elements x and y in any A-algebraic system, i.e. $f(\xi, \eta, \eta) = \xi$ is derived from A.