167. *Another Proof of a Theorem Concerning the Greatest Semilattice-Decomposition of a Semigroup*

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1. **Introduction.** For any semigroup S, consider any congruence ρ on S such that S/ρ is a semilattice, i.e., a commutative idempotent semigroup. Such a ρ is called a semilattice-congruence or simply s-congruence. As is well known, there is the smallest s-congruence ρ_0 on S in the sense of inclusion [1–7]. Let $L=S/\rho_0$ and let S_α, $\alpha \in L$, be a congruence class modulo ρ_0:

$$S=\bigcup_{\alpha\in L} S_\alpha, \quad S_\alpha \cap S_\beta = \square, \quad \alpha \neq \beta.$$

If the cardinal number $|L|$ of L is exactly 1, that is, ρ_0 is the universal relation on S, then S is called s-indecomposable; if $|L|>1$, then S is s-decomposable. The partition of S due to ρ_0 is called the greatest s-decomposition of S, and S/ρ_0 is called the greatest s-homomorphic image of S.

Theorem. *In the greatest s-decomposition of a semigroup S, each congruence class S_α is s-indecomposable.*

This theorem was proved by the author [4] and recently stated by Petrich in [2] without proof. The purpose of this paper is to give a proof of this theorem from somewhat different point of view. Proposition 1 below can be proved by using the above theorem, but here we are going to prove Proposition 1 directly and then to prove the above theorem by using it.

2. **Preliminaries.** Let $a_1, \cdots, a_n$ be elements of a semigroup S. If an element a of S is the product of all of $a_1, \cdots, a_n$ admitting repeated use, then a is said to be fully generated by $a_1, \cdots, a_n$. The set G of all the elements of S which are fully generated by $a_1, \cdots, a_n$ is a non-empty subsemigroup of S. G is called the subsemigroup of S fully generated by $a_1, \cdots, a_n$.

Let $\mathfrak{F}_0$ be the free semigroup generated by n distinct letters $a_1, \cdots, a_n$ in the usual sense, and $\mathfrak{F}$ be the subsemigroup of $\mathfrak{F}_0$ fully generated by $a_1, \cdots, a_n$. $\mathfrak{F}$ is composed of all words any one of which contains all of $a_1, \cdots, a_n$.

Let ρ be any s-congruence on $\mathfrak{F}$. We denote by φ the natural mapping of $\mathfrak{F}$ upon $\mathfrak{F}/\rho$, that is, for $A \in \mathfrak{F}, A\varphi$ of $\mathfrak{F}/\rho$ is the congruence class modulo ρ containing A. For convenience of the