13. A Priori Estimates for Certain Differential Operators with Double Characteristics

By Kenzo SHINKAI

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1. Introduction. In the study of the uniqueness of the Cauchy problem for linear partial differential operators with non-analytic coefficients, Carleman [1] proved a priori estimates of the form

$$(1.1)\qquad \tau^{\gamma}\int |D^{\alpha}u|^2 e^{2\tau\varphi}dx \leqq K\int |P(x, D)u|^2 e^{2\tau\varphi}dx, \quad u\in C_0^{\infty}(\Omega), \tau>\tau_0,$$

where K and γ are constants which are independent of u and τ, and φ is a fixed function. For operators with simple characteristics, many uniqueness theorems have been proved, and it is also known that there exist operators with double complex characteristics for which uniqueness theorems are proved (see Pedersen [8], Mizohata [7], Hörmander [2], Shirota [9], Malgrange [6], Kumano-go [4]).

In this note we shall give a priori estimates of the form (1.1) for the operators of the form

$$(1.2)\qquad P(x, \xi)=P^1(x, \xi)P^2(x, \xi),$$

where P^1 is principally normal and P^2 is elliptic (definitions are given in section 2).

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2. Notations, definitions and theorems from Hörmander. Let α be n-tuples $(\alpha_1, \alpha_2, \cdots, \alpha_n)$ of non-negative integers and we shall use the notations:

$$|\alpha|=\alpha_1+\alpha_2+\cdots+\alpha_n,\ |x|^2=x_1^2+x_2^2+\cdots+x_n^2,\ \xi^{\alpha}=\xi_1^{\alpha_1}\cdot\xi_2^{\alpha_2}\cdots\xi_n^{\alpha_n},$$

$$D_j=-i\frac{\partial}{\partial x_j},\ \text{where } i=\sqrt{-1} \text{ and } D^{\alpha}=D_1^{\alpha_1}\cdot D_2^{\alpha_2}\cdots D_n^{\alpha_n}.$$

Let $P(x, \xi)$ be a polynomial of degree m in n variabils $\xi_1, \xi_2, \cdots, \xi_n$ whose coefficients are functions of x, and denote by $P(x, D)$ the differential operator obtained if ξ_j is replaced by D_j. Let $P_m(x, \xi)$ be the principal part of $P(x, \xi)$ (the homogeneous part of degree m) and we shall also use the notations:

$$P_m^{(j)}(x, \xi)=\frac{\partial}{\partial \xi_j}P_m(x, \xi),\quad P_{m,j}(x, \xi)=\frac{\partial}{\partial x_j}P_m(x, \xi).$$

The following definitions and theorems in this section are quoted from Hörmander [3], Chap. 8.

Definition 1. We shall say that $P(x, D)$ is principally normal in $\bar{\Omega}$ if the coefficients of P_m are in $C^1(\bar{\Omega})$ and there exists a differential