10. On the Absolute Nörlund Summability of a Factored Fourier Series

By Narain Das MEHROTRA

Department of Mathematics, University of Allahabad, India

(Comm. by Kinjirô KUNUGI, M.J.A., Jan. 12, 1965)

1. Let $\sum a_n$ be a given infinite series with the sequence of partial sums $\{s_n\}$. Let $\{p_n\}$ be a sequence of constants, real or complex, and let us write

$$P_n=p_0+p_1+p_2+\cdots+p_n;\ P_{-1}=p_{-1}=0.$$

The sequence to sequence transformation

$$t_n=\frac{1}{P_n}\sum_{\nu=0}^{n} p_{n-\nu}s_\nu=\frac{1}{P_n}\sum_{\nu=0}^{n} p_\nu s_{n-\nu}\quad (P_n\neq 0) \tag{1.1}$$

defines the sequence $\{t_n\}$ of Nörlund mean [3] of the sequence $\{s_n\}$, generated by the sequence of coefficient $\{p_n\}$. The series $\sum a_n$ is said to be absolutely Nörlund summable, or *summable* $|N, p_n|$, if the sequence $\{t_n\}$ is of bounded variation, that is, the series $\sum |t_n-t_{n-1}|$ is convergent [2]. In the special case in which

$$p_n=\frac{1}{n+1}, \tag{1.2}$$

and therefore

$$P_n\sim\log n,\quad \text{as } n\to\infty,$$

the Nörlund mean reduces to the familiar harmonic mean [5]. Thus summability $|N, p_n|$, where p_n is defined as in (1.2), is the same as summability $\left|N, \frac{1}{n+1}\right|$.

Let $f(t)$ be a periodic function with period 2π and integrable (L) over $(-\pi, \pi)$. Without any loss of generality the constant term in the Fourier series can be taken to be zero, that is,

$$\int_{-\pi}^{\pi} f(t)dt=0$$

and

$$f(t)\sim\sum_{n=1}^{\infty}(a_n\cos nt+b_n\sin nt)$$
$$\equiv\sum_{n=1}^{\infty}A_n(t).$$

We write

$$\phi(t)=\frac{1}{2}\{f(x+t)+f(x-t)\},$$

$$\phi_1(t)=\frac{1}{t}\int_0^t\phi(u)du.$$