9. *Two Tauberian Theorems for* (J, p_n) *Summability*

By Kazuo ISHIGURO
Department of Mathematics, Hokkaido University, Sapporo
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§1. The present note is a continuation of a previous paper by the author [4]. We suppose throughout that

$$p_n \geq 0, \qquad \sum_{n=0}^{\infty} p_n = \infty,$$

and that the radius of convergence of the power series

$$p(x) = \sum_{n=0}^{\infty} p_n x^n$$

is 1. Given any series

$$(1) \qquad \sum_{n=0}^{\infty} a_n,$$

with the sequence of partial sums $\{s_n\}$, we shall use the notation:

$$(2) \qquad p_s(x) = \sum_{n=0}^{\infty} p_n s_n x^n.$$

If the series (2) is convergent in the open interval (0, 1), and if

$$\lim_{x \to 1-0} \frac{p_s(x)}{p(x)} = s,$$

we say that the series $\sum_{n=0}^{\infty} a_n$ or the sequence $\{s_n\}$ is summable (J, p_n) to s. As is well known, this method of summability is regular. (See, Borwein [1], Hardy [2], p. 80.) We shall prove, in this note, the following

Theorem 1. *Suppose that*

$$(3) \qquad p_n = O\left(\frac{1}{n}\right)$$

with $p_n > 0$. *Suppose that the series* (1) *is summable* (J, p_n) *to* s, *and that*

$$(4) \qquad a_n = o\left(\frac{p_n}{P_n}\right),$$

where

$$P_n = p_0 + p_1 + \cdots + p_n, \qquad n = 0, 1, \cdots.$$

Then (1) *converges to* s.

Proof. From (3) and (4) we can choose m such that, for $n > m$,

$$(5) \qquad np_n \leq M^{1)}$$

and

1) We use M to denote a constant, possibly different at each occurrence.